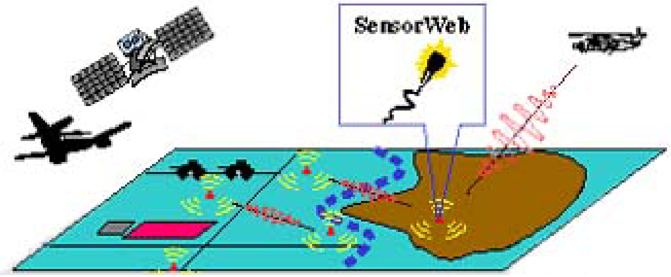

Scalability and information theory for networks with large numbers of nodes

Liang-Liang Xie and P. R. Kumar
Dept. of Electrical and Computer Engineering, and
Coordinated Science Lab
University of Illinois, Urbana-Champaign

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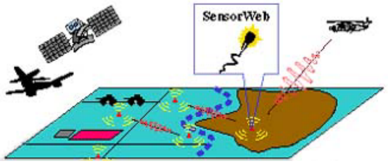
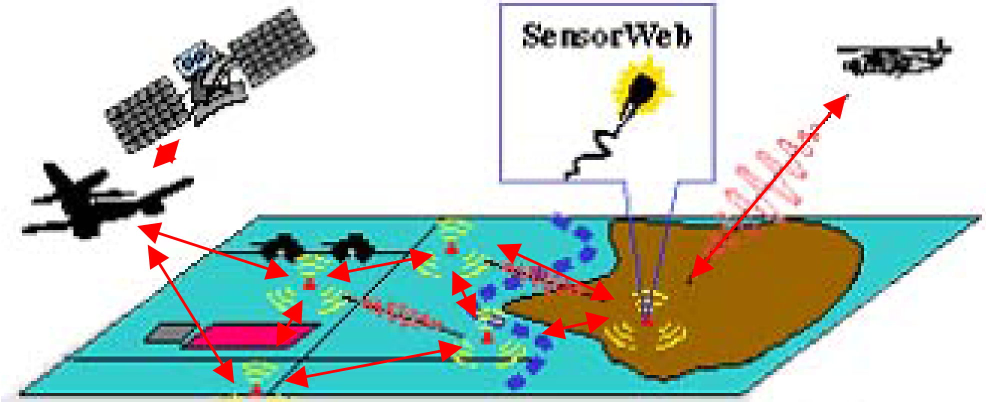
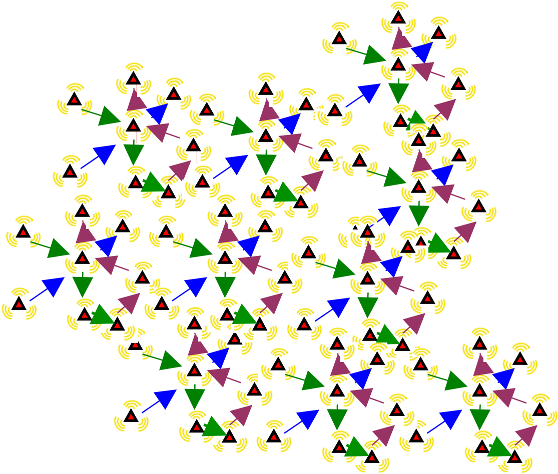
Phone	217-333-7476, 217-244-1653 (Fax)
Email	prkumar@uiuc.edu
Web	http://black.csl.uiuc.edu/~prkumar

Sensor web networks

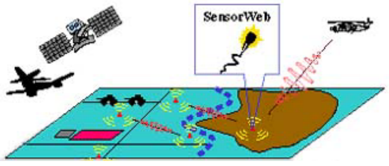


- Networks with large numbers of sensors
  - Potentially large number of information gathering nodes
  - Connected by wireless medium
  - Possibly low power nodes



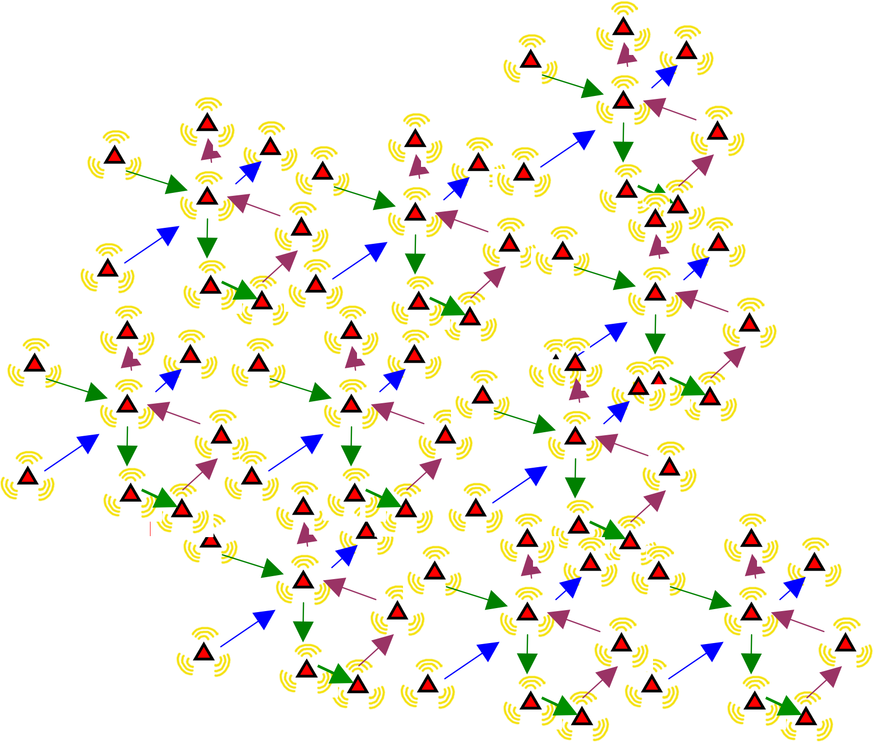


- IT-3: Wireless networks, Network communication and information theory
- RCA 2&3: Fundamental limits on fusion, Network Info Theory, Tradeoffs in local vs. global processing



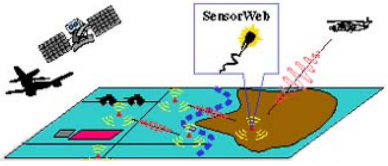
Large wireless networks

- Large wireless networks formed by nodes with radios

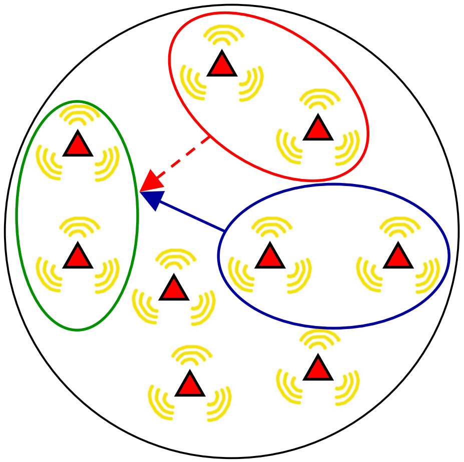


  - There is no *a priori* notion of “links”
  - All nodes simply radiate energy

How should nodes cooperate?

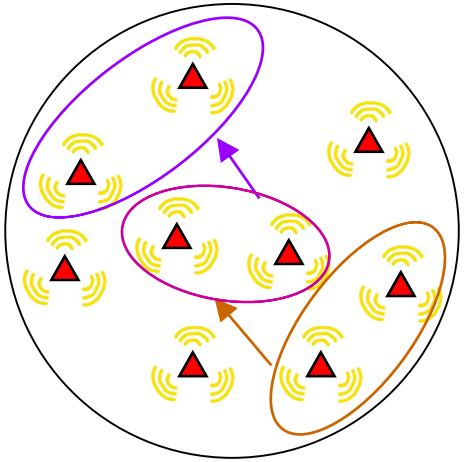


- Nodes can cooperate in complex ways



Nodes in Group A can help cancel the interference of nodes in Group B at nodes in Group C

while

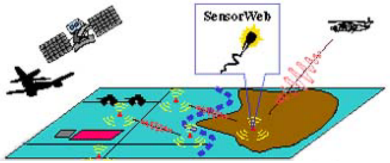


Nodes in Group D coherently transmit to relay packets from Group E to Group F

while **…..** **etc**

  - Very complicated feedback strategies are possible
    - Even notions such as "relaying," "broadcasting," "interference cancellation," "coherent transmission," etc., may be too simplistic
    - The problem has all the complexities of team theory, partially observed systems,etc

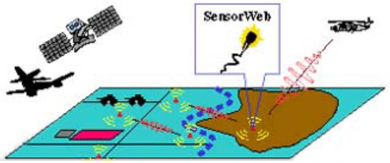
Two fundamental questions



- How should nodes cooperate in maximizing information transfer in a wireless network?
  - The strategy space is infinite dimensional
  - If Information Theory can tell us what the basic strategy should be then we can develop *protocols* to realize the strategy

- How much information can be transported in a wireless network?
  - What are the fundamental limits to information theory?
  - How far is current technology from the optimal?
  - When should we quit trying to do better?

Key Results

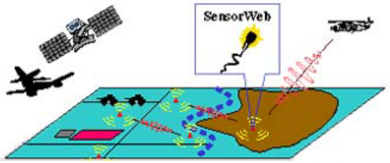


- If there is any absorption in the medium or large attenuation
  - Transport capacity grows like $\Theta(n)$ where n = number of nodes
    - Area grows like $\Omega(n)$
  - Multi-hop operation is optimal

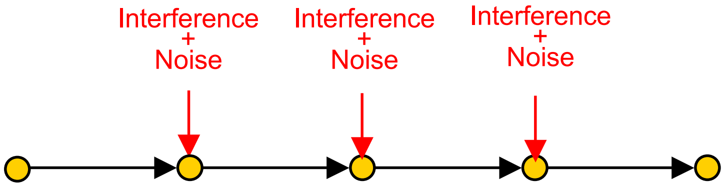
- If there is no absorption, and attenuation is very small
  - Transport capacity can grow like $\Theta(n^\theta)$ for $\theta > 1$
  - Coherent multi-stage relaying with interference cancellation can be optimal

- Along the way
  - Total power used by a network bounds the transport capacity
    - or not
  - A feasible rate for a Gaussian multiple relay channel

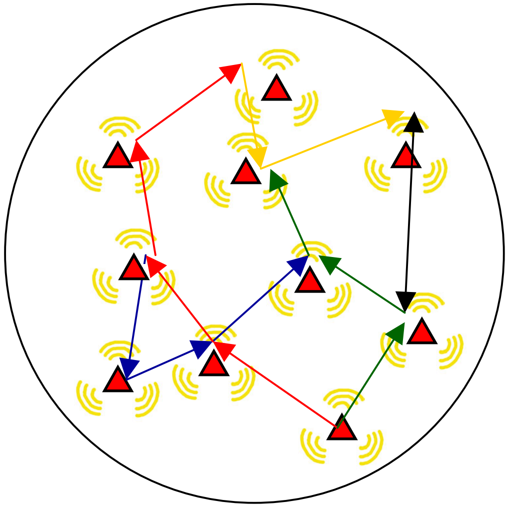
Current proposal: Multi-hop transport

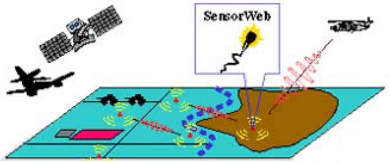


- Multi-hop transport
  - Packets are relayed from node to node
  - A packet is fully decoded at each hop
  - All interference from all other nodes is simply treated as noise



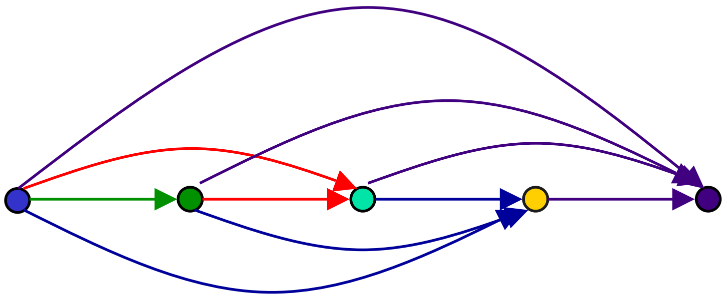
- This choice for the mode of operation gives rise to
  - Routing problem
  - Media access control problem
  - Power control problem
  - ...





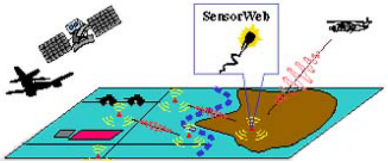
Another strategy

- Coherent multi-stage relaying with interference cancellation



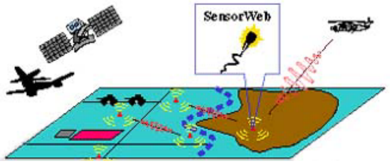
- All upstream nodes coherently cooperate to send a packet to the next node
- A node cancels all the interference caused by all transmissions to its downstream nodes

The Transport Capacity: Definition

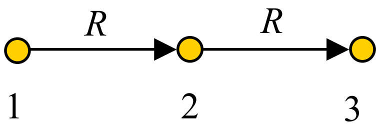


- Source-Destination pairs
  - $(s_1, d_1), (s_2, d_2), (s_3, d_3), \ldots, (s_l, d_l)$
- Distances
  - $\rho_1, \rho_2, \rho_3, \ldots, \rho_l$ distances between the sources and destinations
- Feasible Rates
  - $(R_1, R_2, R_3, \ldots, R_l)$ feasible rates for these source-destination pairs
- Distance-weighted sum of rates
  - $\Sigma_i R_i \rho_i$
- <u>Transport Capacity</u>
  - $C_T = \sup \Sigma_i R_i \rho_i$
    - Supremum is taken over all feasible rates $(R_1, R_2, R_3, \ldots, R_l)$

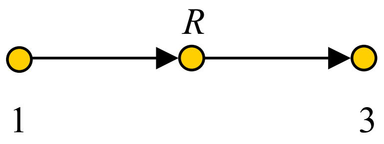
The Transport Capacity



- $C_T = \sup \Sigma_i R_i\rho_i$
  - Measured in bit-meters/second or bit-meters/slot
  - Analogous to man-miles/year considered by airlines
  - Upper bound to what network can carry
    - irrespective of which sources, destinations and their rates
  - Satisfies a scaling law
    - Conservation law which restricts what network can provide
    - Irrespective of whether it is of prima facie interest
  - However it is also of natural interest
  - Allows us to compare apples with apples

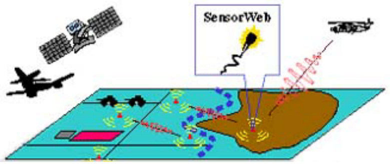


$= (R, R, 0)$ or

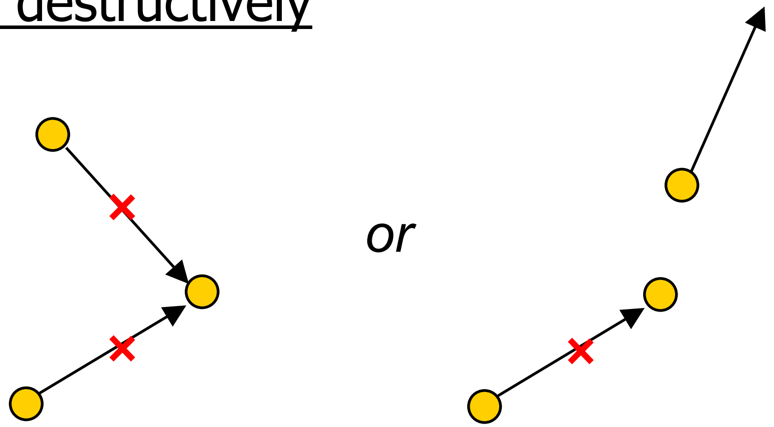


$= (0, 0, R)$

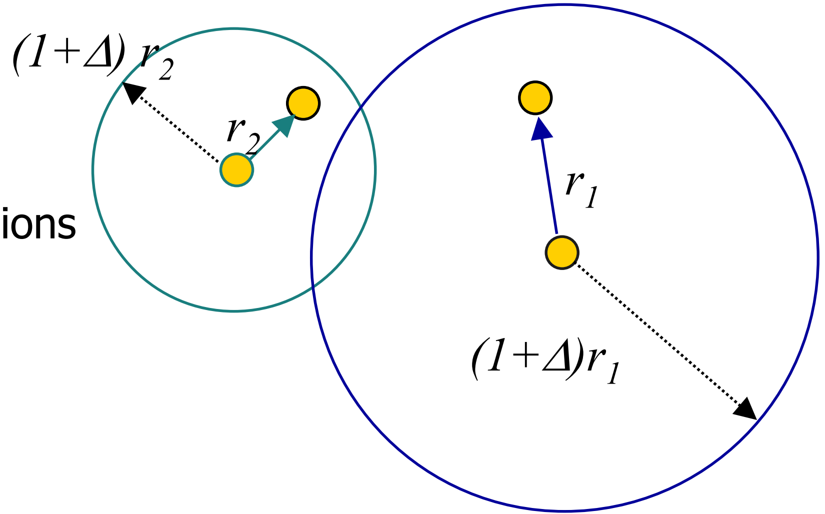
Models where packets "collide"



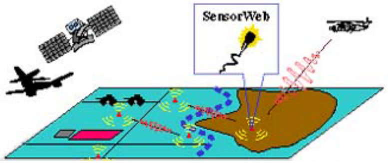
- In some technologies - Packets collide destructively
  - Example: If all interference is regarded as noise



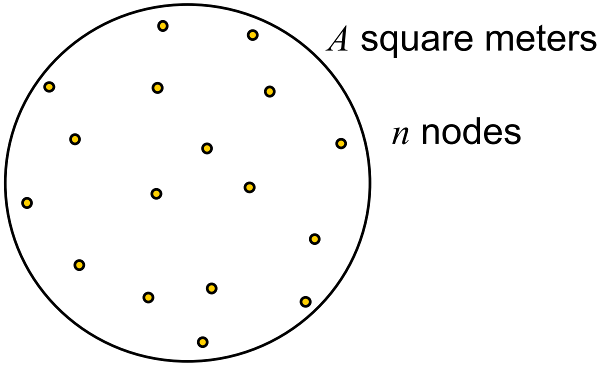
- Gupta-Kumar model
  - Reception is successful if
    - Receiver not in vicinity of two transmissions
    - Or SINR > β
    - Or Rate depends on SINR



Scaling laws under "collision" model

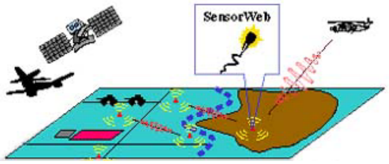


- Theorem (Gupta-Kumar 2000)
  - Disk of area A square meters
  - n nodes
  - Each can transmit at W bits/sec

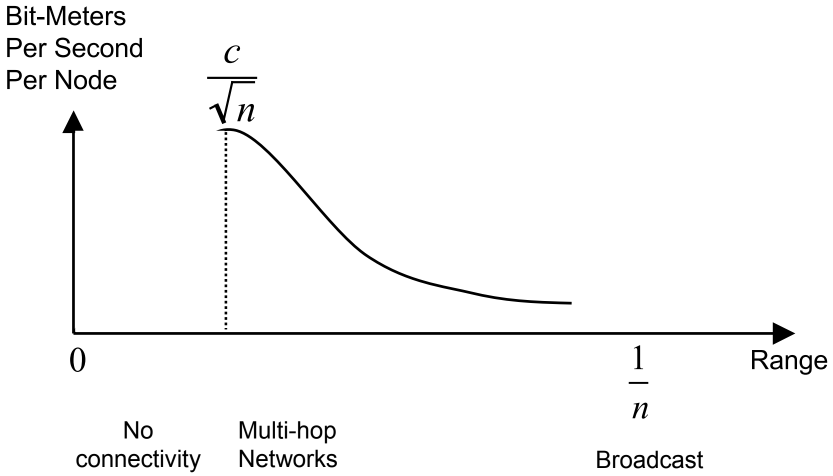


- Best Case: Network can transport $\Theta\left(W\sqrt{An}\right)$ bit-meters/second
- Random case: Each node can obtain a throughput $\Theta\left(\frac{1}{\sqrt{n\log n}}\right)$ bits/second
- Square root law
  - Transport capacity doesn't increase linearly, but only like square-root
  - Each node gets $\frac{c}{\sqrt{n}}$ bit-meters/second

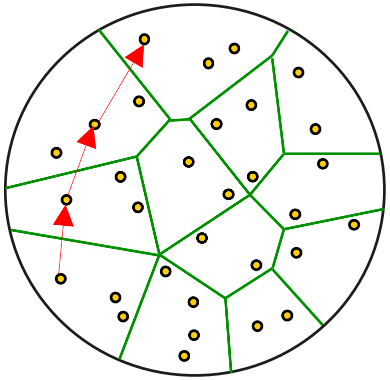
Optimal operation under “collision” model



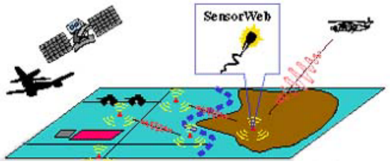
- Optimal operation
  - Multi-hop is optimal

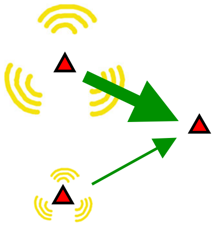


- Optimal multi-hop architecture
  - Group nodes into cells of size log n
  - Common power level for all nodes is nearly optimal
  - Power should be as small as possible subject to network connectivity
  - Just enough power to reach all points in neighboring cell
  - Can route packets along nearly straight line path from cell to cell

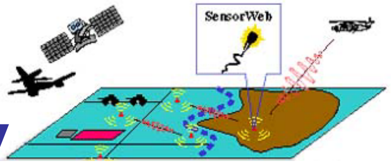


But interference is not interference

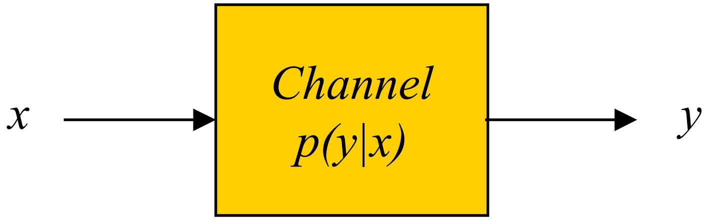


- 
  - Receiver can first decode loud signal perfectly
  - Then subtract the loud signal
  - Then decode the soft signal perfectly
  - So excessive interference can be very good
  - Packets do <u>not</u> destructively collide
- Interference is information!
- So we need an information theory for networks to determine
  - How to operate wireless networks
  - How much information wireless networks can transport

Shannon’s Information Theory



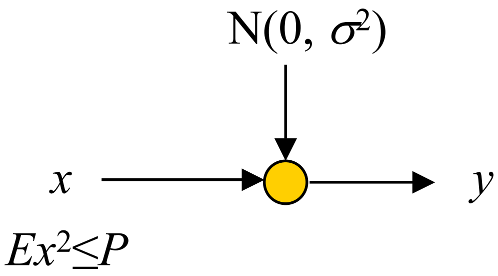
- Shannon’s Capacity Theorem
  - Channel Model $p(y|x)$
    - Discrete Memoryless Channel

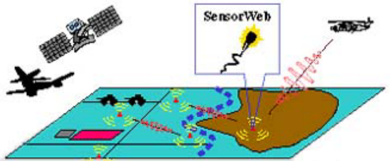


  - Capacity = $\text{Max}_{p(x)} \mathrm{I}(X;Y)$ bits/channel use

$$I(X;Y) = \sum_{x,y} p(x,y) \log\left(\frac{p(X,Y)}{p(X)p(Y)}\right)$$

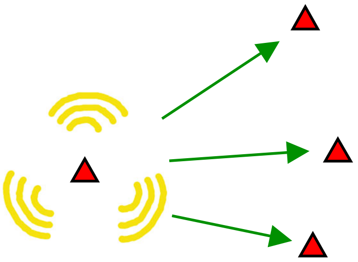
- Additive White Gaussian Noise (AWGN) Channel
  - Capacity = $S\left(\frac{P}{\sigma^2}\right)$, where $S(z) = \frac{1}{2}\log(1+z)$



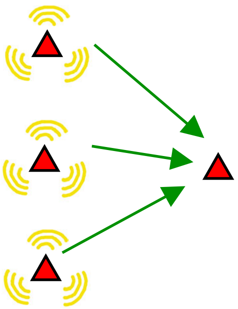


Network information theory: The triumphs

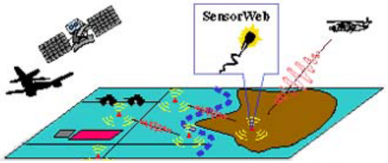
- Gaussian broadcast channel



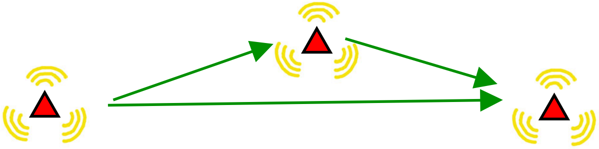
- Gaussian multiple access channel



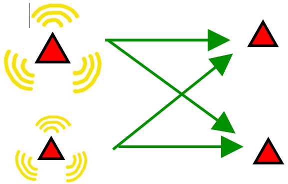
Network information theory: The unknowns



- The simplest relay channel

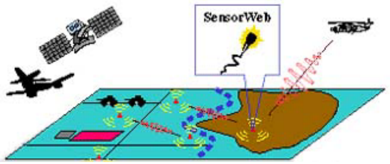


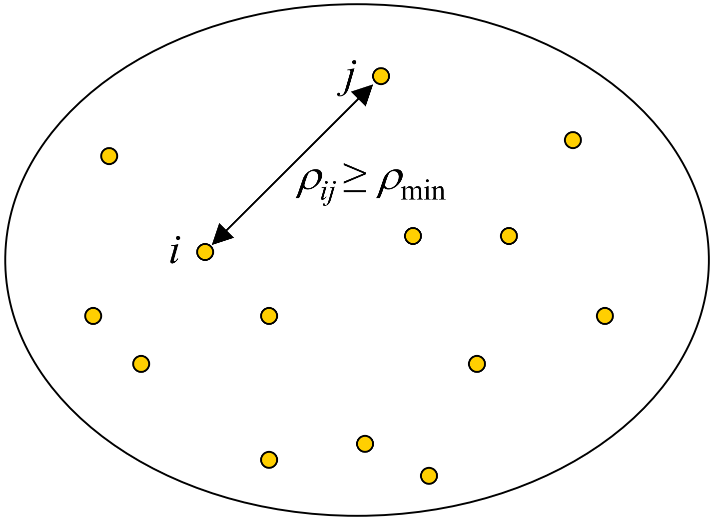
- The simplest interference channel



- Systems being built are much more complicated and the possible modes of cooperation can be much more sophisticated
  - How to analyze?
  - Need a large scale information theory

Model of system: A planar network

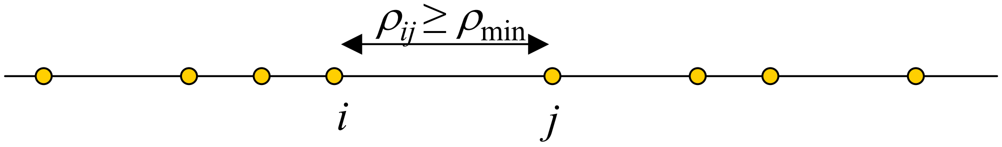




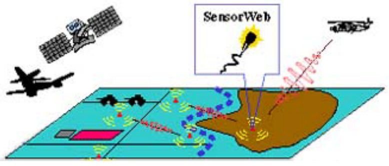
- n nodes in a plane
- ρ_{ij} = distance between nodes i and j
- Minimum distance between nodes

$$\rho_{ij} \geq \rho_{\min} > 0$$

- Or a linear network



Model of signal attenuation

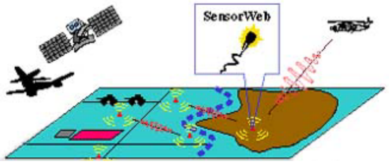


- Signal path loss attenuation with distance:

Attenuation over a distance $\rho = \dfrac{e^{-\gamma\rho}}{\rho^{\delta}}$

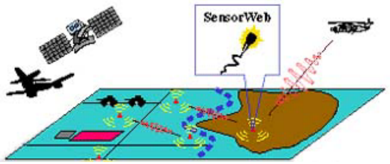
  - ρ = distance between transmitter and receiver
  - $\gamma \geq 0$ is the absorption constant
    - Loss of $20\gamma \log_{10} e$ db per meter
  - Generally $\gamma > 0$ since the medium is absorptive unless over a vacuum
  - $\delta > 0$ is the path loss exponent

The Results



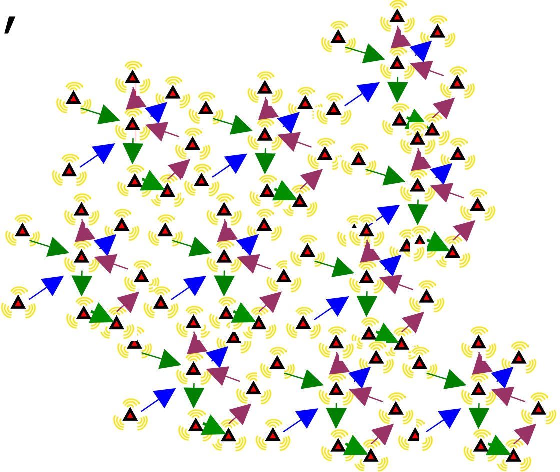
When there is absorption or a large path loss

The total power bounds the transport capacity



- Theorem
  - Suppose $\gamma > 0$, I.e., there is some absorption,

    or $\delta > 3$ if there is no absorption at all
  - Then for all Planar Networks



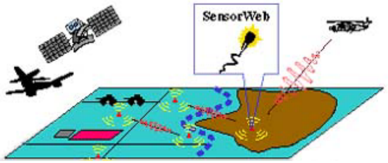
$$C_T \leq \frac{c_1(\gamma, \delta, \rho_{\min})}{\sigma^2} \cdot P_{total}$$

where

$$c_1(\gamma, \delta, \rho_{\min}) = \frac{2^{2\delta+7}}{\gamma^2 \rho_{\min}^{2\delta+1}} \frac{e^{-\gamma\rho_{\min}/2}(2 - e^{-\gamma\rho_{\min}/2})}{(1 - e^{-\gamma\rho_{\min}/2})} \qquad \text{if } \gamma > 0$$

$$= \frac{2^{2\delta+5}(3\delta - 8)}{(\delta-2)^2(\delta-3)\rho_{\min}^{2\delta-1}} \qquad \text{if } \gamma = 0 \text{ and } \delta > 3$$

O(n) upper bound on Transport Capacity



- Theorem
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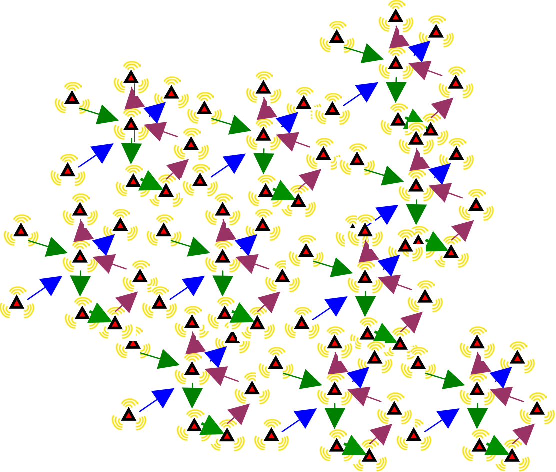


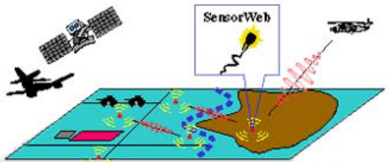
$$C_T \leq \frac{c_1(\gamma, \delta, \rho_{\min}) P_{ind}}{\sigma^2} \cdot n$$

where

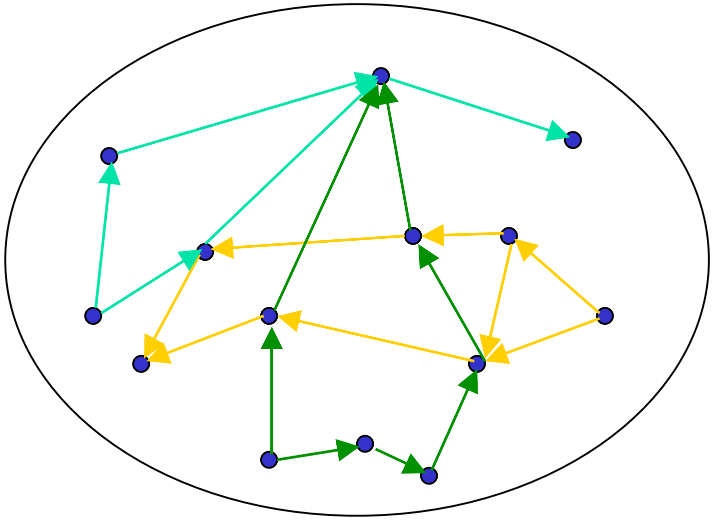
$$c_1(\gamma, \delta, \rho_{\min}) = \frac{2^{2\delta+7}}{\gamma^2 \rho_{\min}^{2\delta+1}} \frac{e^{-\gamma\rho_{\min}/2}(2 - e^{-\gamma\rho_{\min}/2})}{(1 - e^{-\gamma\rho_{\min}/2})} \qquad \text{if } \gamma > 0$$

$$= \frac{2^{2\delta+5}(3\delta - 8)}{(\delta - 2)^2 (\delta - 3) \rho_{\min}^{2\delta-1}} \qquad \text{if } \gamma = 0 \text{ and } \delta > 3$$

What can multihop transport achieve?



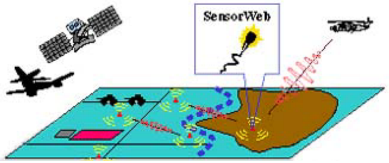
- Theorem
  - A set of rates (R_1, R_2, … , R_l) can be supported by multi-hop transport if
  - Traffic can be routed, possibly over many paths, such that
  - No node has to relay more than $S\left(\frac{e^{-2\gamma\bar{\rho}}P_{ind}/\bar{\rho}^{2\delta}}{c_3(\gamma,\delta,\rho_{\min})P_{ind}+\sigma^2}\right)$
  - where $\bar{\rho}$ is the longest distance of a hop



and $$c_3(\gamma,\delta,\rho_{\min}) = \frac{2^{3+2\delta}e^{-\gamma\rho_{\min}}}{\gamma\rho_{\min}^{1+2\delta}} \qquad \text{if } \gamma > 0$$

$$= \frac{2^{2+2\delta}}{\rho_{\min}^{2\delta}(\delta-1)} \qquad \text{if } \gamma = 0 \text{ and } \delta > 1$$

Multihop transport can achieve $\Theta(n)$



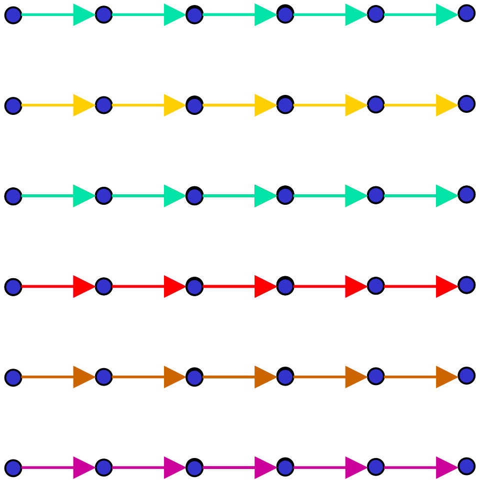
- <u>Theorem</u>
  - Suppose $\gamma > 0$, there is some absorption,
  - Or $\delta > 1$, if there is no absorption at all
  - Then in a regular planar network

$$C_T \geq S\left(\frac{e^{-2\gamma} P_{ind}}{c_2(\gamma,\delta) P_{ind} + \sigma^2}\right) \cdot n$$

where

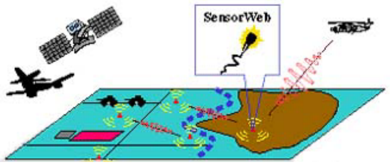
$$c_2(\gamma,\delta) = \frac{4(1+4\gamma)e^{-2\gamma} - 4e^{-4\gamma}}{2\gamma(1-e^{-2\gamma})} \qquad \text{if } \gamma > 0$$

$$= \frac{16\delta^2 + (2\pi - 16)\delta - \pi}{(\delta-1)(2\delta-1)} \qquad \text{if } \gamma = 0 \text{ and } \delta > 1$$



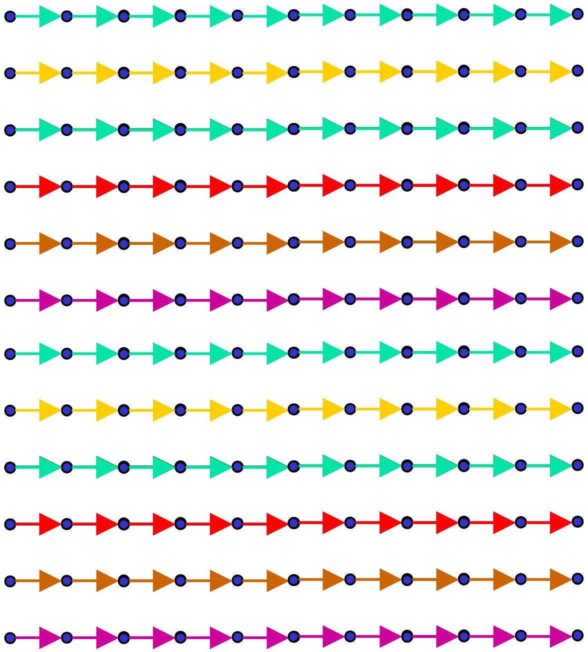
$\sqrt{n}$ sources each sending over a distance $\sqrt{n}$

Optimality of multi-hop transport

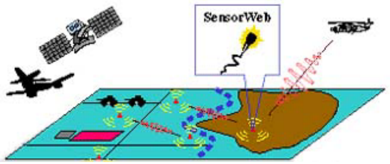


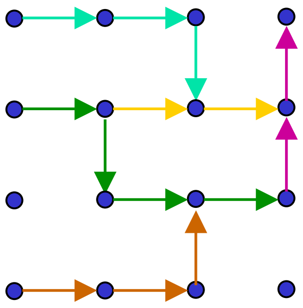
- Corollary
  - So if $\gamma > 0$ or $\delta > 3$
  - And multi-hop achieves $\Theta(n)$
  - Then it is optimal with respect to the transport capacity
    - at least up to order

Example

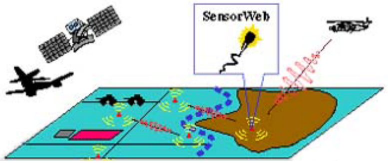


Multi-hop is almost optimal in a random network



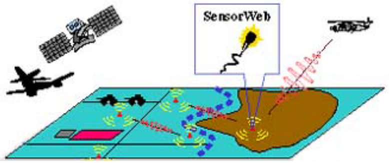


- Theorem
  - Consider a regular planar network
  - Suppose each node randomly chooses a destination
    - Choose a node nearest to a random point in the square
  - Suppose $\gamma > 0$ or $\delta > 1$
  - Then multihop can provide $\Omega\left(\frac{1}{\sqrt{n \log n}}\right)$ bits/time-unit for every source with probability →1 as the number of nodes n $\rightarrow \infty$
- Corollary
  - Nearly optimal since transport achieved is $\Omega\left(\frac{n}{\sqrt{\log n}}\right)$



What happens when the attenuation is very low?

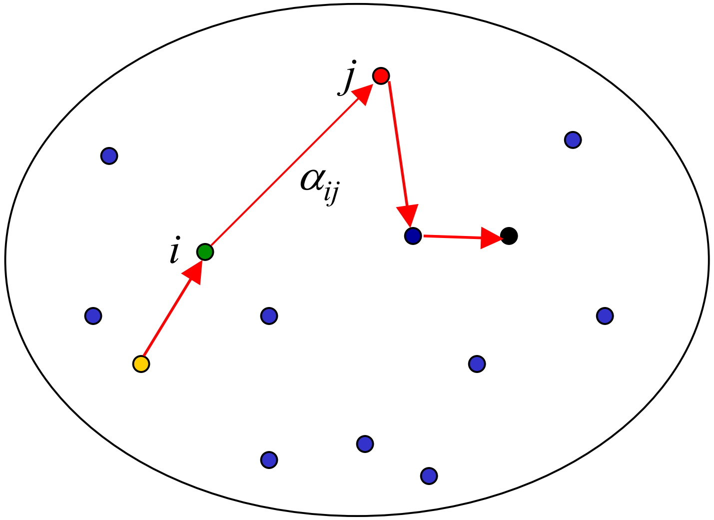
A feasible rate for Gaussian multiple-relay channel

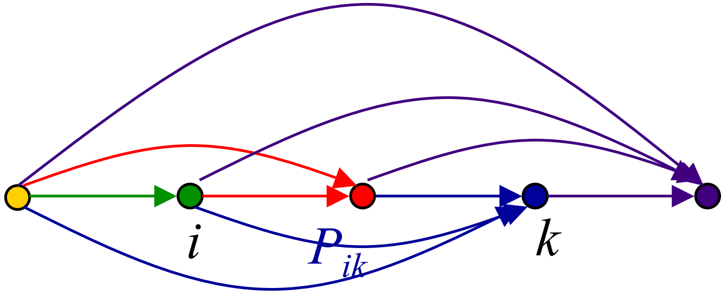


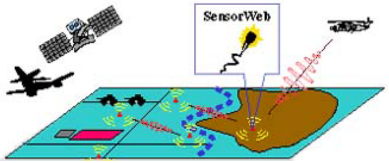
- Theorem
  - Suppose α_{ij} = attenuation from i to j
  - Choose power P_{ik} = power used by i intended directly for node k
  - where $\sum_{k=i}^{M} P_{ik} \le P_i$
  - Then

$$R < \min_{1 \le j \le n} S\left(\frac{1}{\sigma^2} \sum_{k=1}^{j} \left(\sum_{i=0}^{k-1} \alpha_{ij} \sqrt{P_{ik}} \right)^2 \right)$$

is feasible

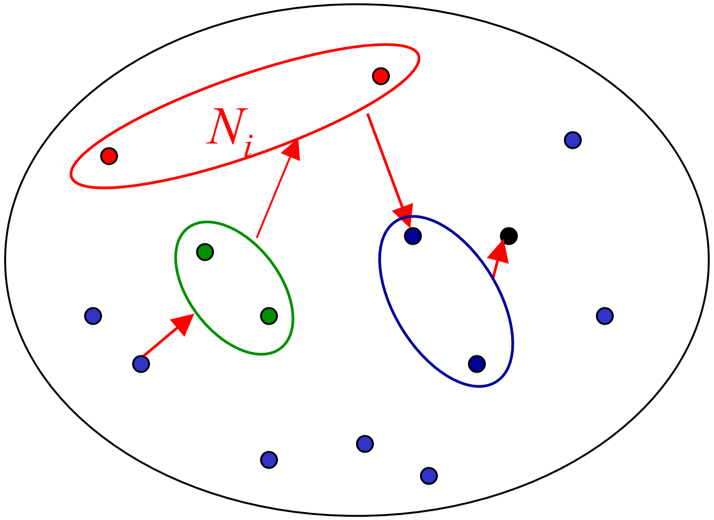






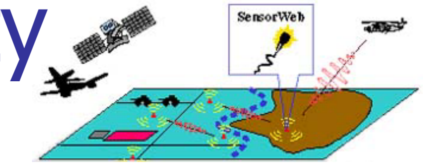
A group relaying version

- Theorem
  - A feasible rate for group relaying



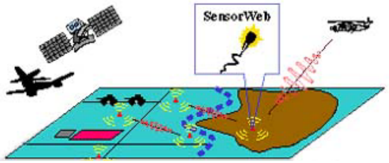
- $$R < \min_{1\le j\le M} S\left(\frac{1}{\sigma^2}\sum_{k=1}^{j}\left(\sum_{i=0}^{k-1}\alpha_{N_iN_j}\sqrt{P_{ik}/n_i}\cdot n_i\right)^2\right)$$

Unbounded transport capacity for fixed total power



- Theorem
    - If $\gamma > 0$, there is no absorption at all,
    - And $\delta < 3/2$
    - Then C_T can be unbounded in regular planar networks even for fixed P_{total}
- Theorem
    - If $\gamma > 0$ and $\delta < 1$ in regular planar networks
    - Then no matter how many many nodes there are
    - No matter how far apart the source and destination are chosen
    - A fixed rate R_{min} can be provided for the single-source destination pair

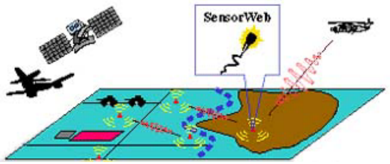
Networks with transport capacity $\Theta(n^{\theta})$



- Theorem
  - Suppose $\gamma = 0$
  - For every $1/2 < \delta < 1$, and $1 < \theta < 1/\delta$
  - There is a family of linear networks with

    $$C_T = \Theta(n^{\theta})$$

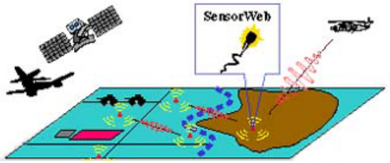
  - The optimal strategy is coherent multi-stage relaying with interference cancellation



Concluding Remarks

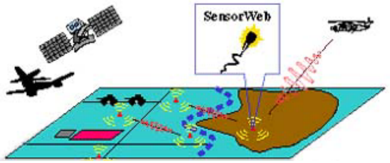
- Studied networks with arbitrary numbers of nodes
  - Explicitly incorporated distance in model
    - Distances between nodes
    - Attenuation as a function of distance
- Make progress by asking for less
  - Instead of studying capacity region, study the transport capacity
  - Instead of asking for exact results, study the scaling laws
    - The exponent is more important
    - The preconstant is also important but is secondary - so bound it
  - Draw some broad conclusions
    - Optimality of multi-hop when absorption or large path loss
    - Optimality of coherent multi-stage relaying with interference cancellation when no absorption and very low path loss
- Open problems abound
  - What happens for intermediate path loss when there is no absorption
  - The channel model is simplistic

U. S. Army interactions



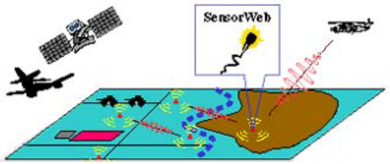
- Panel Member, Triennial Research Strategy Planning Workshop U.S. Army Research Office, Computing and Information Sciences Division, Charleston, SC, Jan 3-5, 2001.
- Board of Visitors of the U.S. Army Research Office, 6.1 Mathematical Sciences Program Review, May 21, 2001, Research Triangle Park

Other events

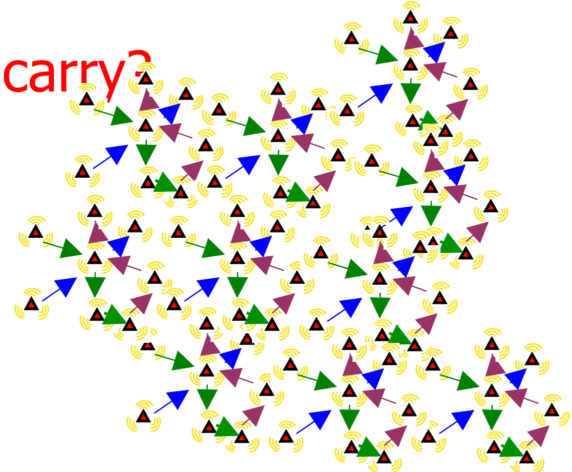


- **Plenary Talks**
  - Plenary Talk, SIAM Annual Meeting, July 9-13, 2001, San Diego
  - Keynote speaker, ITCOM + OPTICOMM 2001: The Convergence of Information Technologies and Communications, Denver, August 19-24, 2001.
  - Plenary Lecture, The Fifth Stochastik-Tage: German Open Conference on Probability and Statistics, Magdeburg, Germany, March 19-22, 2002
  - Plenary Talk, 10th Mediterranean Conference on Control and Automation, Lisbon, Portugal, July 9-13, 2002.
- **Invited Talks**
  - Conference on Stochastic Networks, June 19-24, 2000, University of Wisconsin, Madison
  - NSF/ONR Workshop on Cross-Layer Design in Adaptive Ad Hoc Networks: From Signal Processing to Global Networking, May 31-June 1, 2001, Cornell University
  - Symposium on Complex Systems Modeling and Optimization in the Information Age To Celebrate 45 Years of Outstanding Contribution of Prof. Yu-Chi "Larry" Ho, June 23-24, 2001, Harvard University
  - NSF Workshop on an Infrastructure for Mobile and Wireless Systems , The Convergence of Information Technologies and Communications, Scottsdale, Arizona, Oct. 15, 2001.
  - Stochastic Theory and Control Workshop, University of Kansas, Oct. 18-20, 2001.
  - Massively Distributed Self-Organizing Networks , May 13-17, 2002. Institute for Pure and Applied Mathematics, University of California, Los Angeles
  - Conference on Stochastic Networks, June 24-29, 2002, Stanford University
- **Others**
  - Panel, Future Directions in Control and Dynamical Systems, June 16-17, 2000
  - Chair and Organizer, Workshop on Wireless Networks, Institute for Mathematics and its Applications, Minneapolis, August 8-10, 2001.
  - Illinois/Berkeley Student Workshop on Wireless Networks and Convergence, , CSL, Urbana, November 17-18, 2001
  - European Wireless Conference, Florence, Italy, Feb 26-28, 2002.

Issues: Analysis, Protocols and Architecture of convergence

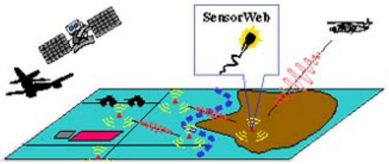


- How much information can wireless networks carry?



- How should ad hoc networks be operated?
  - Design of operating protocols which adapt to the environment
- Towards convergence of communication, computing. sensing and actuation
  - Abstractions and Architecture

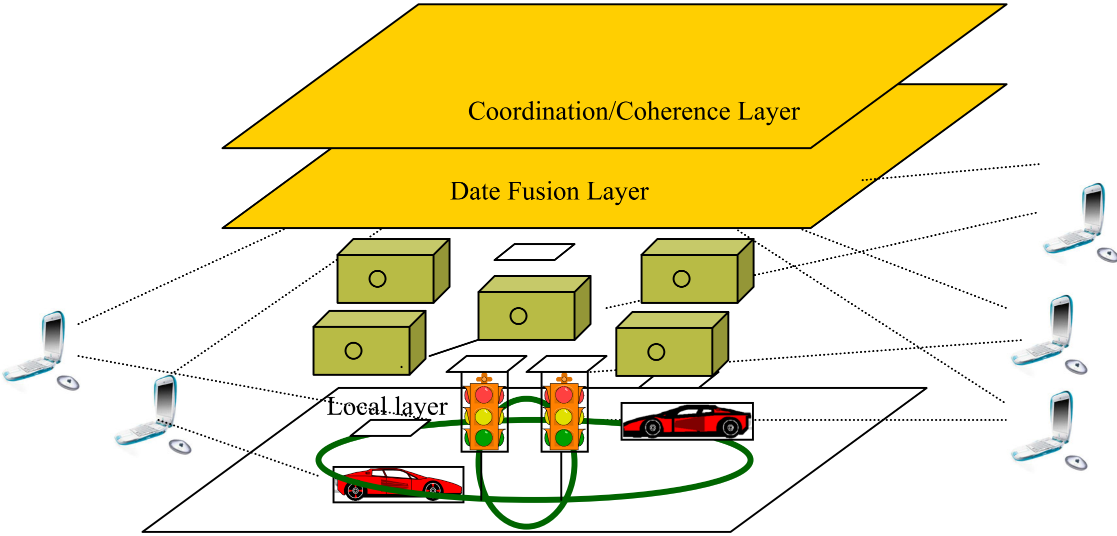
An experimental testbed for networking sensors



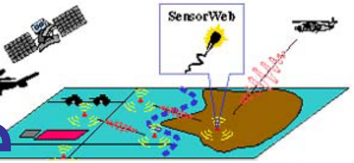
- Next step in IT revolution: Convergence of communication, computing, and control
- Sensors and actuators galore communicating over wireless and interacting with physical world
- Issue: How do we organize such distributed real-time systems?
    - Eg. If traffic lights and cars and sensors can talk to each other, how would you architect the system?

What are the right abstractions?
What is the architecture?

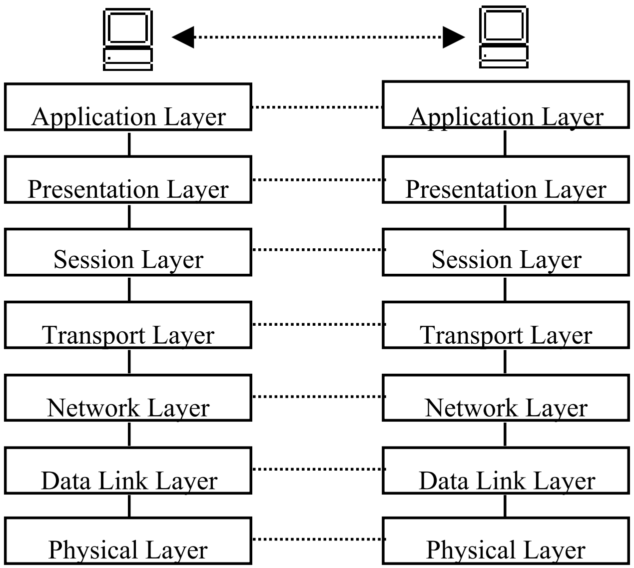
- A testbed for convergence at Univ of Illinois
- Layer of interest for this project
    - Sensing, Networking, Data Fusion layers



The importance of architecture



- Success of Internet is due to its architecture
  - Notion of peer-to-peer protocols
  - Hierarchy of layers
  - Allows plug-and-play
  - Proliferation of technology



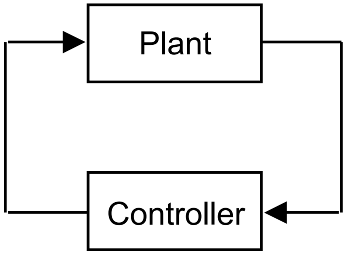
Application Layer | Application Layer
Presentation Layer | Presentation Layer
Session Layer | Session Layer
Transport Layer | Transport Layer
Network Layer | Network Layer
Data Link Layer | Data Link Layer
Physical Layer | Physical Layer

- Success of serial computing
  - von Neumann bridge (Valiant)
  - Hardware designers and software designers need only to conform to abstractions of each other

Hardware

Software

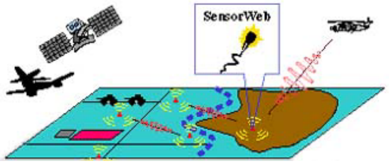
- Control system paradigm
  - Plant and controller separation



Plant

Controller

To obtain paper



- Papers can be downloaded from

  http://black.csl.uiuc.edu/~prkumar

- For hard copy send email to

  prkumar@uiuc.edu