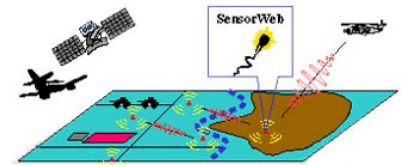


# Data Association/Fusion for Heterogenous Sensors in Nonlinear and Dispersive Media

John Fisher

SensorWeb MURI Review Meeting

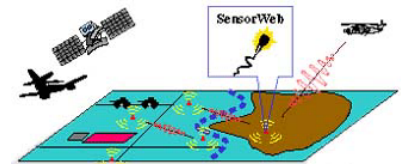
June 14, 2002



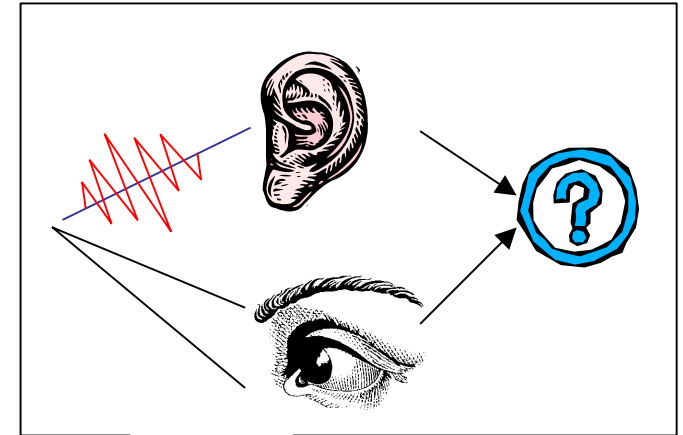
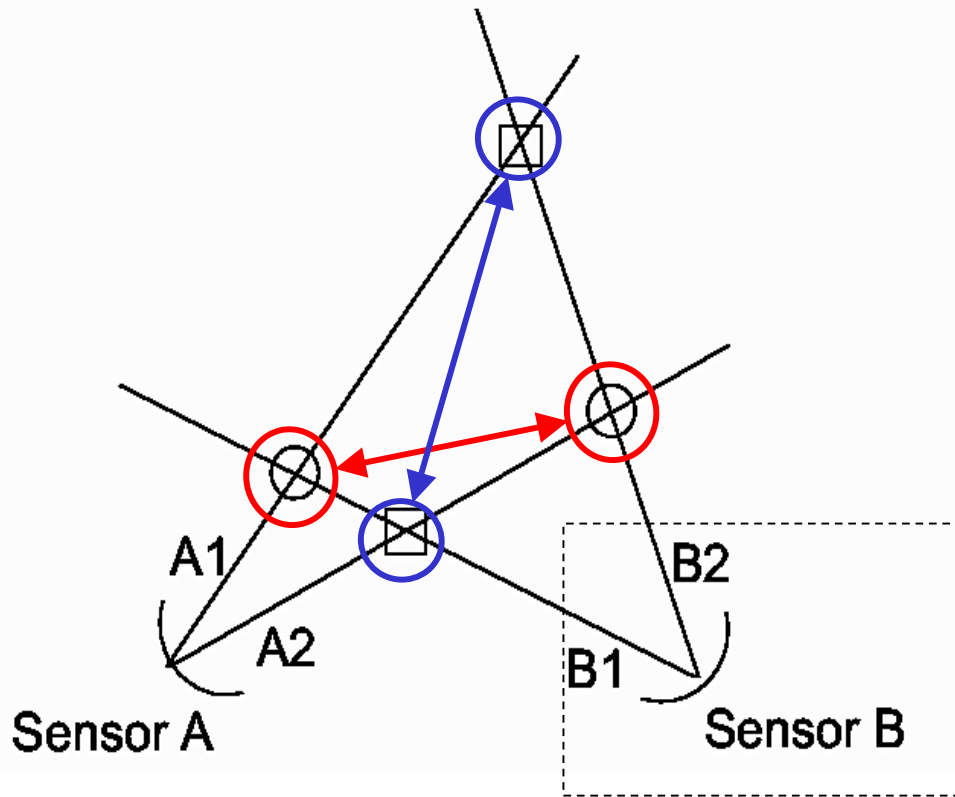
# Goals

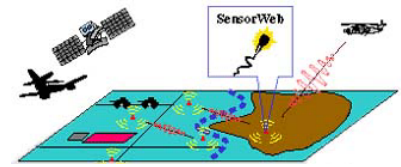
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- Data association across nonlinear and/or dispersive channels
- Statistical models of heterogeneous sensors

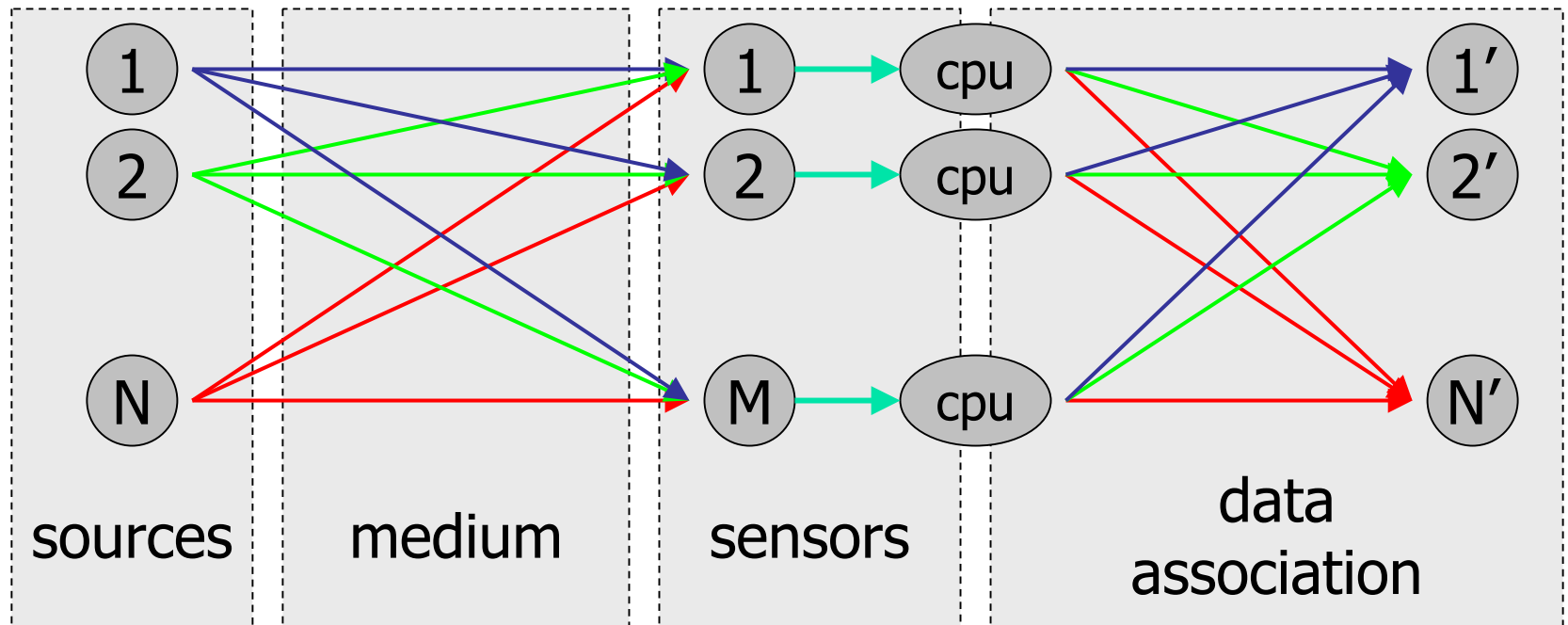


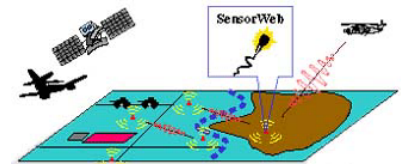
# Data Association and Fusion





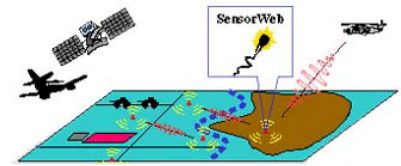
# Fusion and Data Association





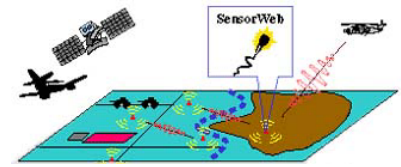
# Issues and Challenges

- In the absence of a **prior statistical model** and given nonlinear and/or dispersive media
- How do we
    - solve data association problem in a principled manner?
    - fuse/utilize multi-modal data?
  - Can we
    - propagate the results to higher level algorithms in the form of likelihoods or scores?

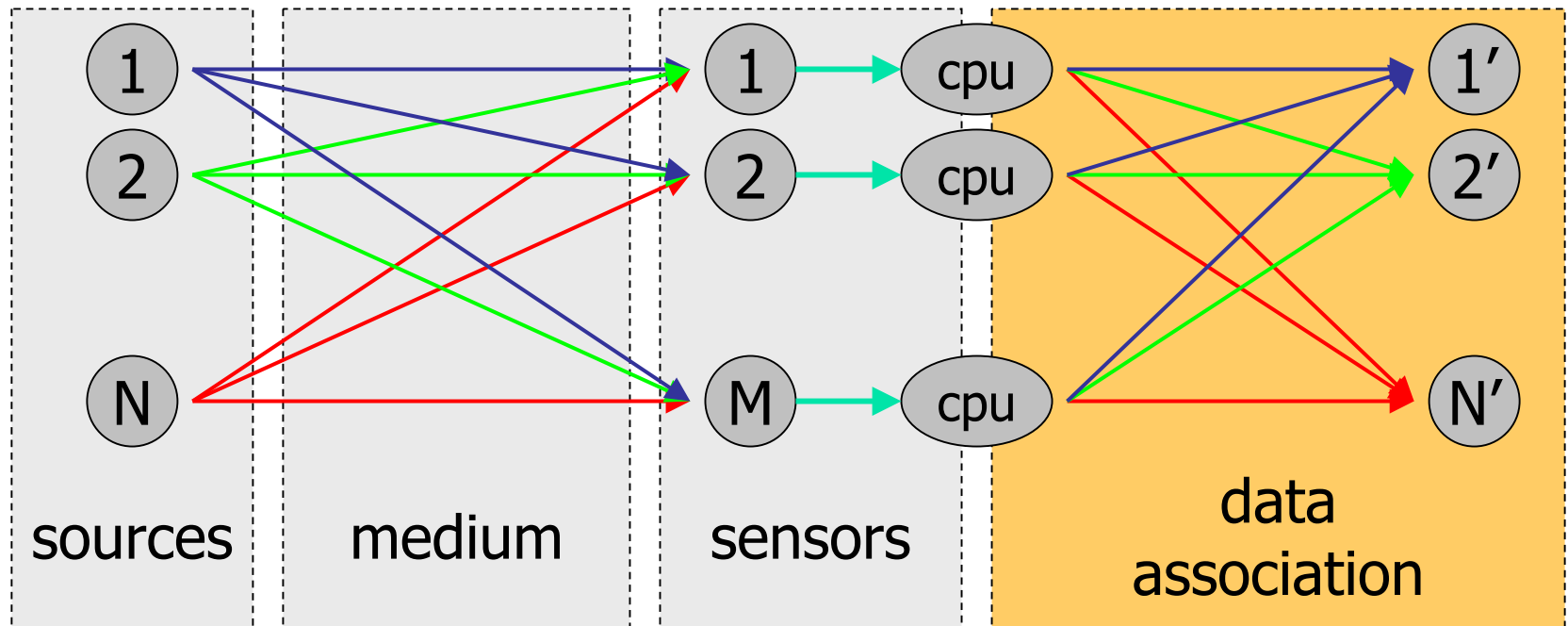


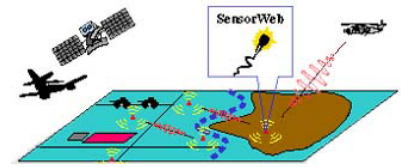
# Vital Statistics

- IT-2 (fusion of heterogeneous sensors in unstructured and uncertain environments)
- RCA-1 (self-calibration)
- RCA-5 (fusion algorithms)
- RCA-7 (data for experiments and demos)
- Ties to RCA-2&3 (Tradeoffs in local vs. global processing)
- Contributors
  - Fisher, Ihler, Çetin, Willsky
- Preliminary outputs
  - Several publications and talks
  - A number of academic, industrial, and DoD interactions



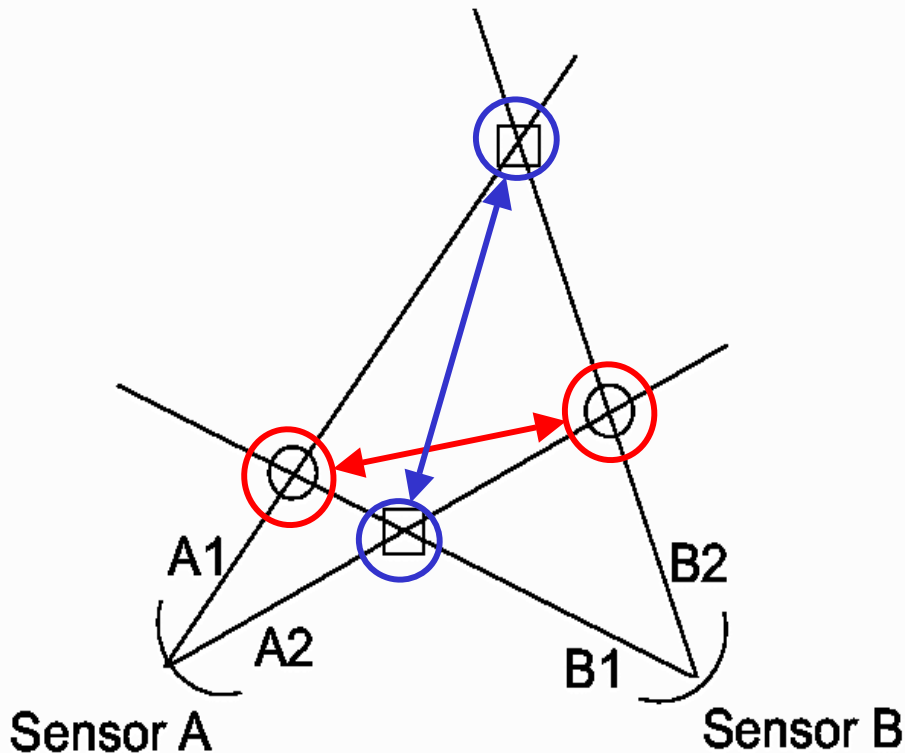
# Data Association



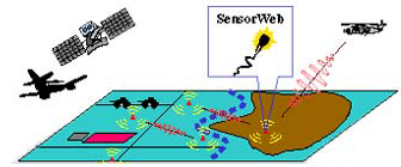


# Data Association Problem

- Sensors receive data and associated bearing estimates
- Bearing alone results in an inherent ambiguity
- Correspondence problem between A1,2 and B1,2







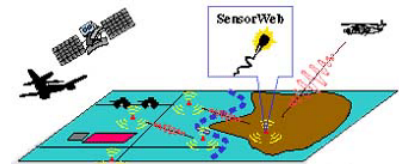
# Data Association as a Hypothesis Testing Problem

Typically, one incorporates factorization *and* an assumed parameterized density under either hypothesis

$$\begin{aligned}
 H_1 : [A_1, A_2, B_1, B_2]_i &\square p_{H_1}(A_1, A_2, B_1, B_2) = p_{H_1}(A_1, B_1) p_{H_1}(A_2, B_2) \\
 H_2 : [A_1, A_2, B_1, B_2]_i &\square p_{H_2}(A_1, A_2, B_1, B_2) = p_{H_2}(A_1, B_2) p_{H_2}(A_2, B_1)
 \end{aligned}$$

resulting in a log-likelihood of

$$\begin{aligned}
 \log L &= \sum_i \log \left( \frac{p_{H_1}([A_1, B_1]_i) p_{H_1}([A_2, B_2]_i)}{p_{H_2}([A_1, B_2]_i) p_{H_2}([A_2, B_1]_i)} \right) \\
 &= \sum_i \log \left( \frac{p_{H_1}([A_1, B_1]_i) p_{H_1}([A_2, B_2]_i) p(A_{1i}) p(A_{2i}) p(B_{1i}) p(B_{2i})}{p(A_{1i}) p(A_{2i}) p(B_{1i}) p(B_{2i}) p_{H_2}([A_1, B_2]_i) p_{H_2}([A_2, B_1]_i)} \right)
 \end{aligned}$$



# Data Association as a Hypothesis Testing Problem

Expectation (or limit) under  $H_1$  yields

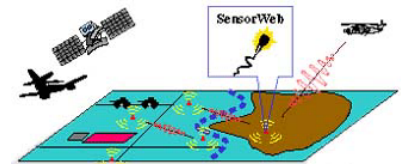
$$E_{H_1} \{ \log L \} \propto I(A_1; B_1) + I(A_2; B_2) + D(p_{H_1}(A_1) p_{H_1}(B_2) \| p_{H_2}([A_1, B_2])) + D(p_{H_1}(A_2) p_{H_1}(B_1) \| p_{H_2}([A_2, B_1]))$$

Terms related to association (i.e. statistical dependency)

under  $H_2$  yields

$$E_{H_2} \{ \log L \} \propto -I(A_1; B_2) - I(A_2; B_1) - D(p_{H_2}(A_1) p_{H_2}(B_1) \| p_{H_1}([A_1, B_1])) - D(p_{H_2}(A_2) p_{H_2}(B_2) \| p_{H_1}([A_2, B_2]))$$

Terms related to density modeling assumptions

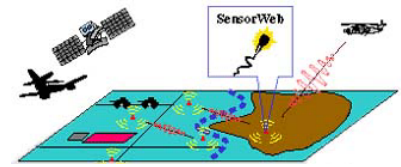


## What if prior models are not available?

- Suppose we have “perfect” density estimates from the measurements.
- We can estimate the factorization under  $H_1$  and  $H_2$  from data, but the terms due to the assumed density go away.
- Data association becomes a hypothesis test over factorizations

$$E_{H_1} \{ \log L \} \propto I(A_1, B_1) + I(A_2, B_2)$$
$$E_{H_2} \{ \log L \} \propto - (I(A_1, B_2) + I(A_2, B_1))$$

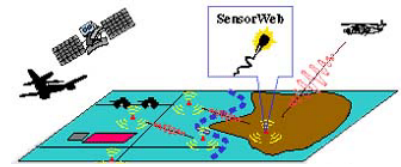
- So we lose the benefit of parametric density terms, but ...
- We also don't suffer when the density model terms are wrong.



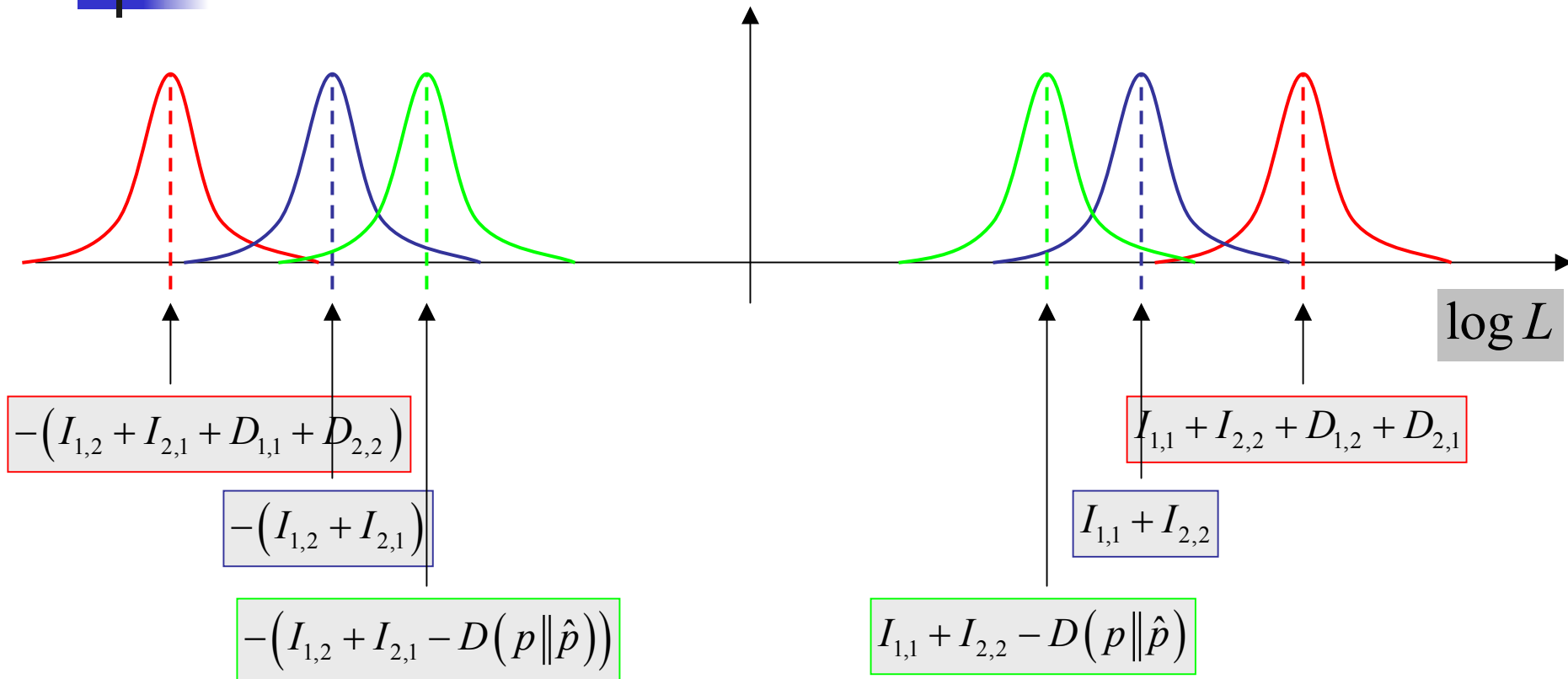
## Density Estimation Effects

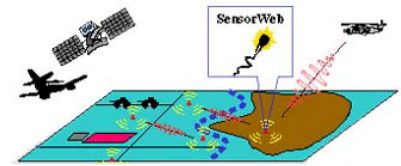
- Of course, we have imperfect density estimates introducing additional biases in our log-likelihood computation.

$$\begin{aligned}
 E_{H_1} \{ \log L \} &\propto I(A_1, B_1) + I(A_2, B_2) - \\
 &\quad D\left(p(A_1, B_1) p(A_2, B_2) \parallel \hat{p}(A_1, B_1) \hat{p}(A_2, B_2)\right) + \\
 &\quad D\left(p(A_1) p(A_2) p(B_1) p(B_2) \parallel \hat{p}(A_1, B_2) \hat{p}(A_2, B_1)\right) \\
 E_{H_2} \{ \log L \} &\propto -\left(I(A_1, B_2) + I(A_2, B_1) - \right. \\
 &\quad \left. D\left(p(A_1, B_2) p(A_2, B_1) \parallel \hat{p}(A_1, B_2) \hat{p}(A_2, B_1)\right) + \right. \\
 &\quad \left. D\left(p(A_1) p(A_2) p(B_1) p(B_2) \parallel \hat{p}(A_1, B_1) \hat{p}(A_2, B_2)\right)\right)
 \end{aligned}$$



# Distribution over log-likelihoods



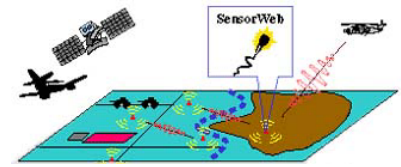


# Dimensionality – Feature Extraction

- High dimensionality precludes direct density estimation.
- Dependence may be captured in a low-dimensional subspace (or manifold).
- Compromise by projecting to a lower dimensional space.

$$\begin{aligned} H_1 &: [A_1, A_2, B_1, B_2] \rightarrow [f(A_1; \alpha_{11}), f(A_2; \alpha_{21}), f(B_1; \beta_{11}), f(B_2; \beta_{21})] \\ H_2 &: [A_1, A_2, B_1, B_2] \rightarrow [g(A_1; \alpha_{12}), g(A_2; \alpha_{22}), g(B_1; \beta_{12}), g(B_2; \beta_{22})] \end{aligned}$$

- Use kernel density estimator to compensate for complexity in the feature space.
- Objective is to do this **without** training.



# Dimensionality – Feature Extraction

- By choosing the projection coefficients to maximize MI under the hypothesis we minimize the deviation of the LRT in the feature space from the LRT in the measurement space

$$\sum_i \log \left( \frac{p_{H_1}([A_1, B_1]_i) p_{H_1}([A_2, B_2]_i)}{p_{H_2}([A_1, B_2]_i) p_{H_2}([A_2, B_1]_i)} \right)$$

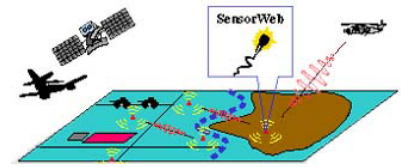
$$H_1 : \arg \max_{\alpha_{11}, \alpha_{21}, \beta_{11}, \beta_{21}} [I(f(A_1; \alpha_{11}); f(B_1; \beta_{11})) +$$

$$I(f(A_2; \alpha_{21}); f(B_2; \beta_{21}))]$$

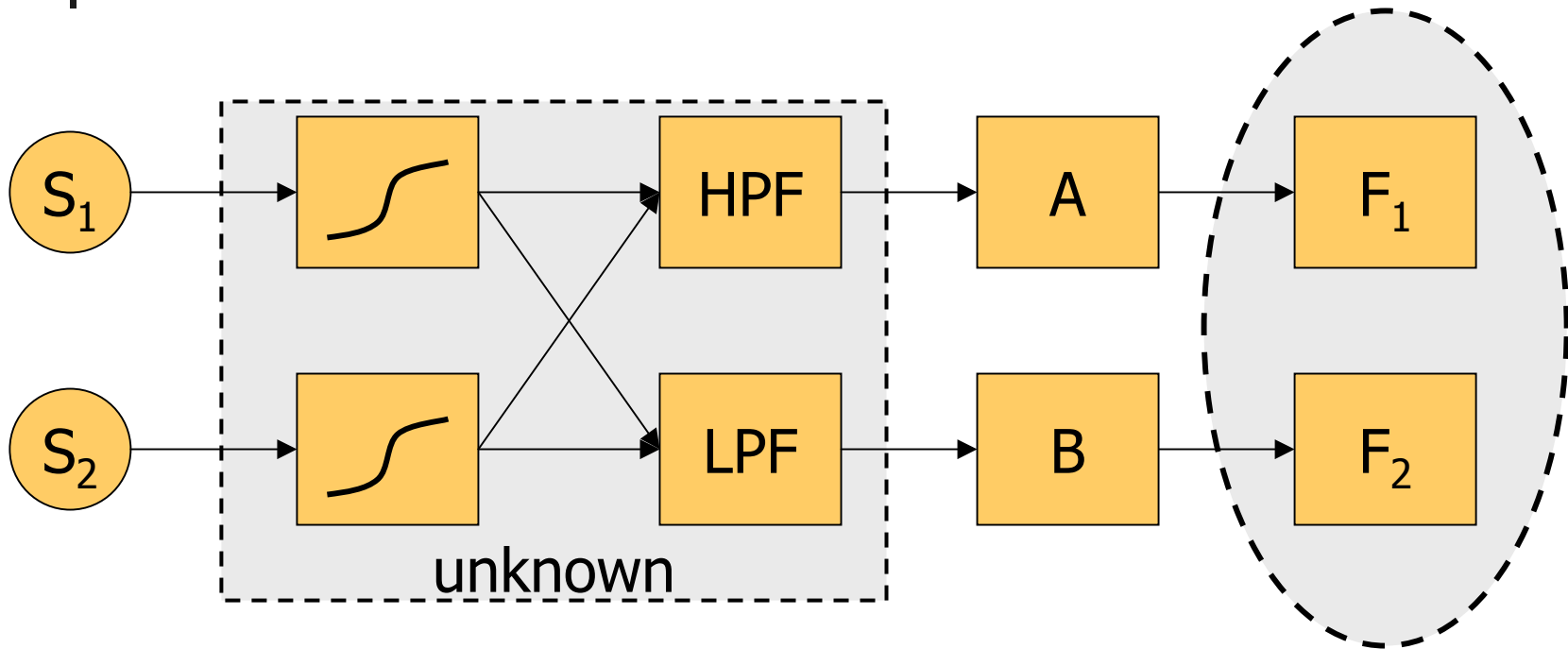
$$H_2 : \arg \max_{\alpha_{12}, \alpha_{22}, \beta_{12}, \beta_{22}} [I(g(A_1; \alpha_{12}); g(B_2; \beta_{22})) +$$

$$I(g(A_2; \alpha_{22}); g(B_1; \beta_{12}))]$$

$$\sum_i \log \left( \frac{\tilde{p}_{H_1}([f(A_1; \alpha_{11}), f(B_1; \beta_{11})]_i) \tilde{p}_{H_1}([f(A_2; \alpha_{21}), f(B_2; \beta_{21})]_i)}{\tilde{p}_{H_2}([g(A_1; \alpha_{12}), g(B_2; \beta_{22})]_i) \tilde{p}_{H_2}([g(A_2; \alpha_{22}), g(B_1; \beta_{12})]_i)} \right)$$

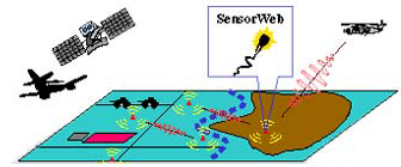


# Nonlinear Channel



Received signals are uncorrelated but *not* independent



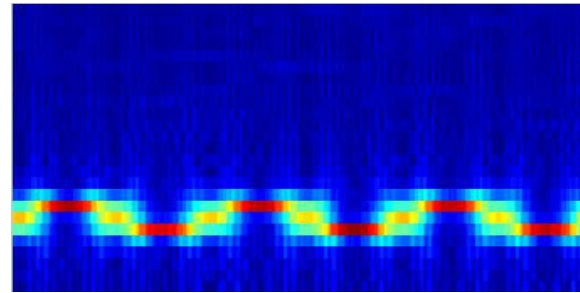
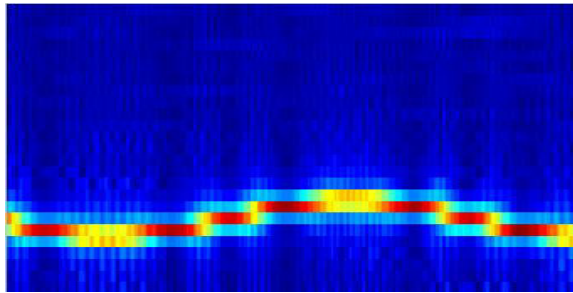


# Narrowband, Uncorrelated Signals

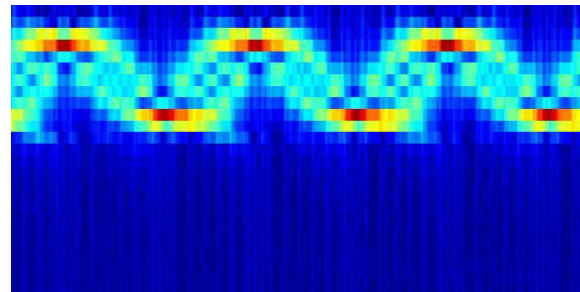
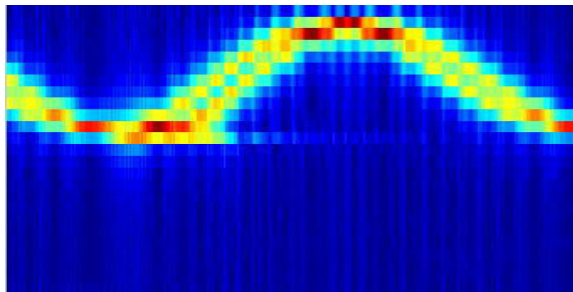
Signal 1

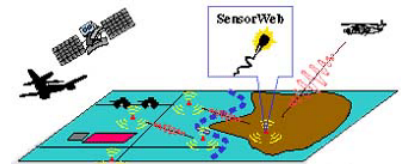
Signal 2

Sensor A

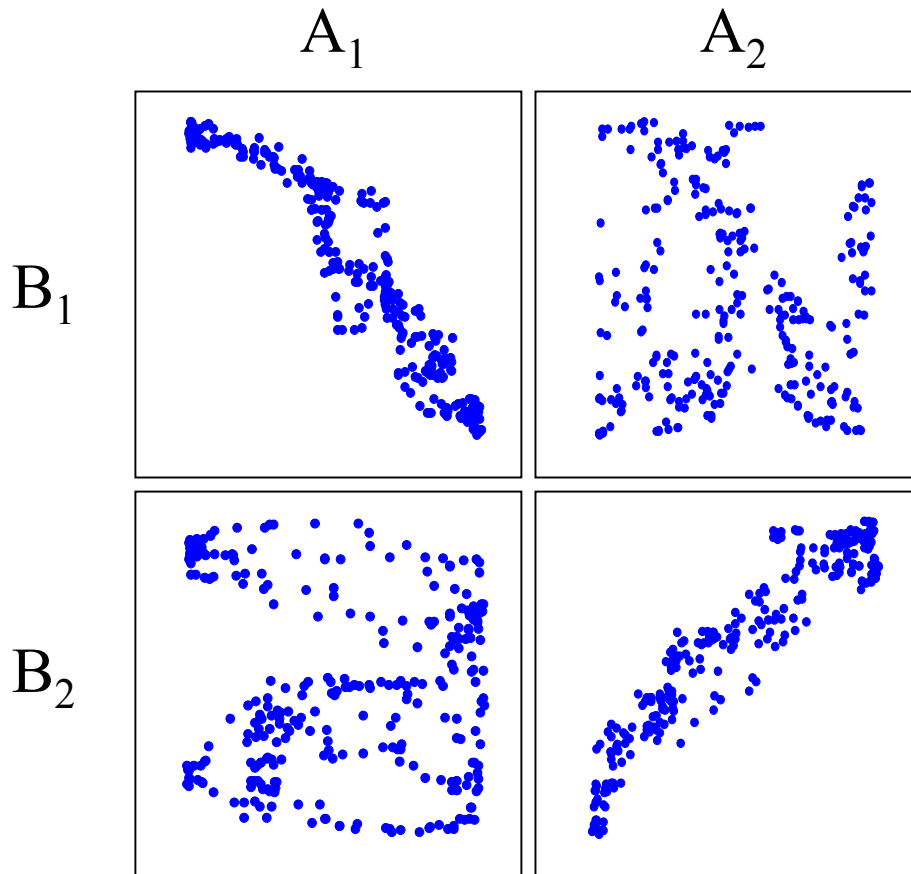


Sensor B

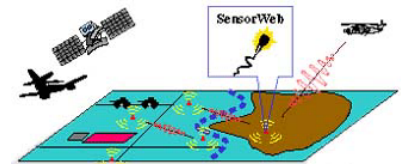




# Feature Space



- Spectra projected down to single scalar value.
- Density estimated in MI optimized feature space.
- MI (data association log-likelihood) computed over feature space.
- Dependence is clear.

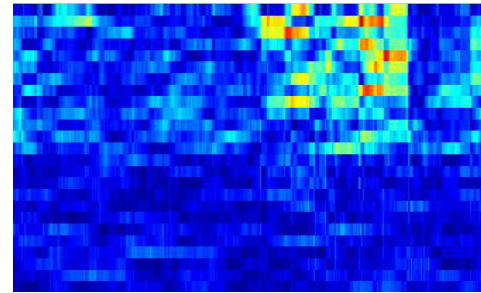
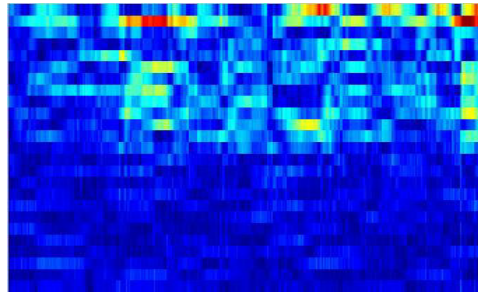


# Wideband, Uncorrelated Signals

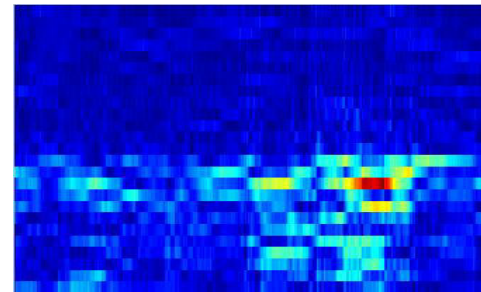
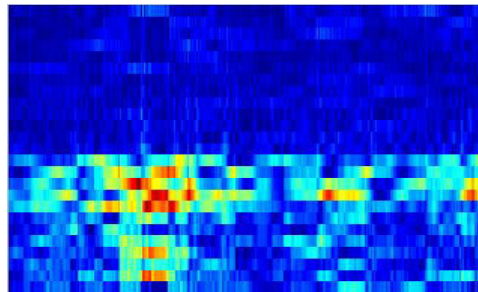
Signal 1

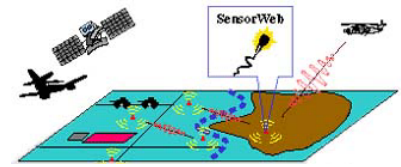
Signal 2

Sensor A

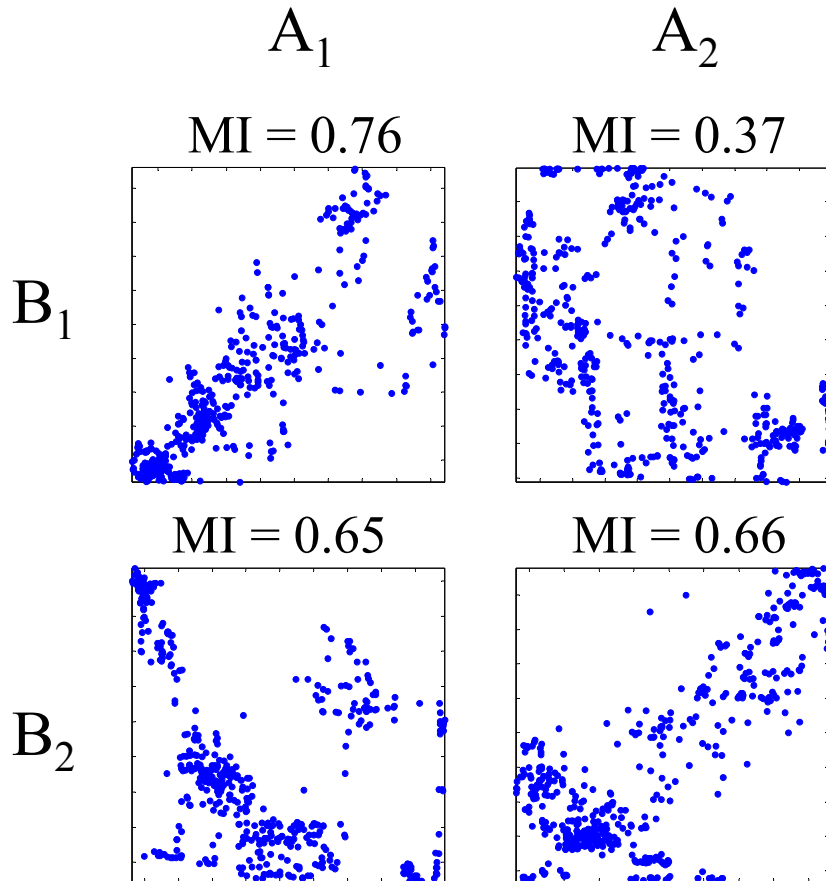


Sensor B



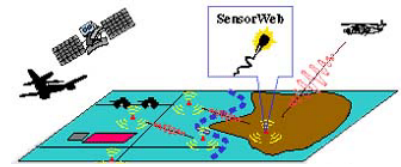


# Feature Space

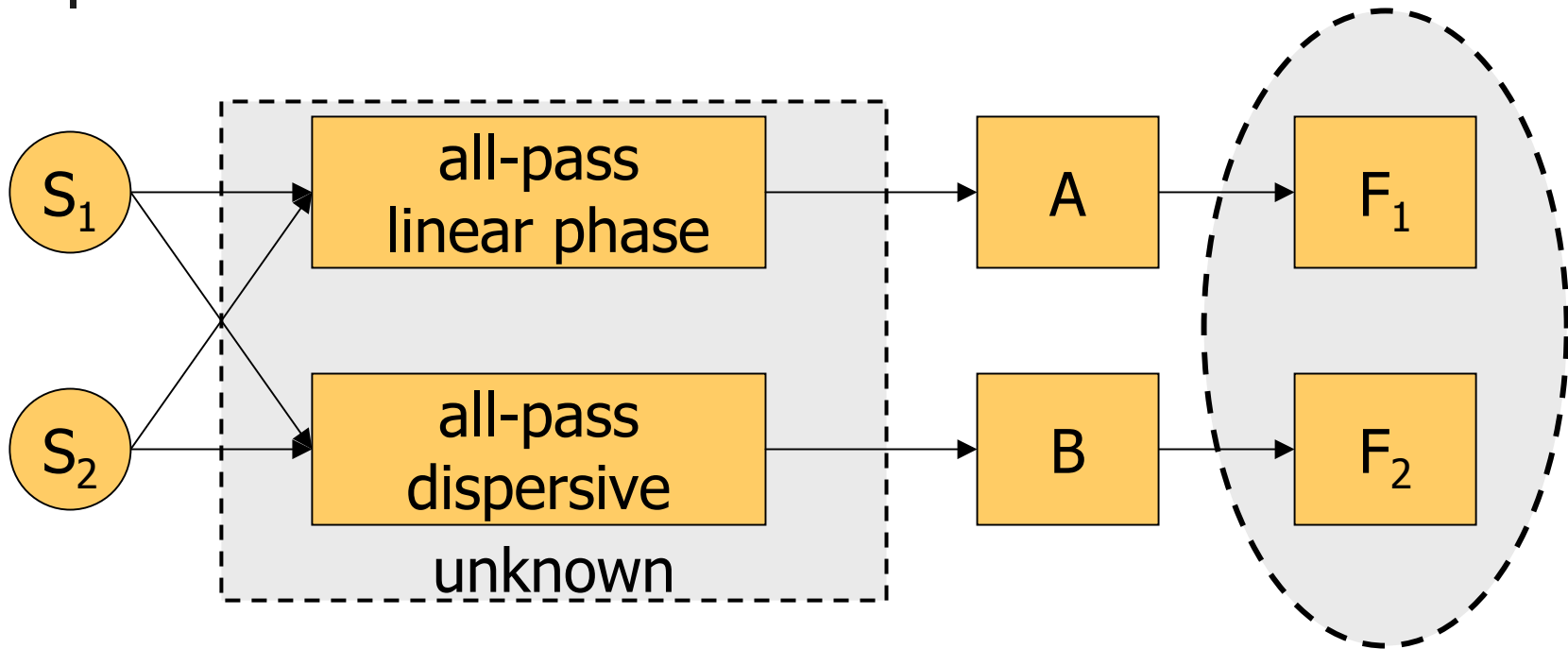


- Association is less obvious for wideband case.
- This is also reflected in MI values.
- Combined score still chooses correct association

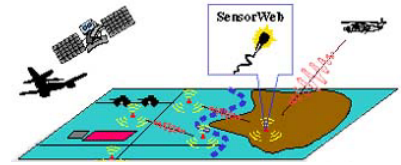
$$0.143 (1-1,2-2) > 0.102 (1-2,2-1)$$



# Dispersive Medium

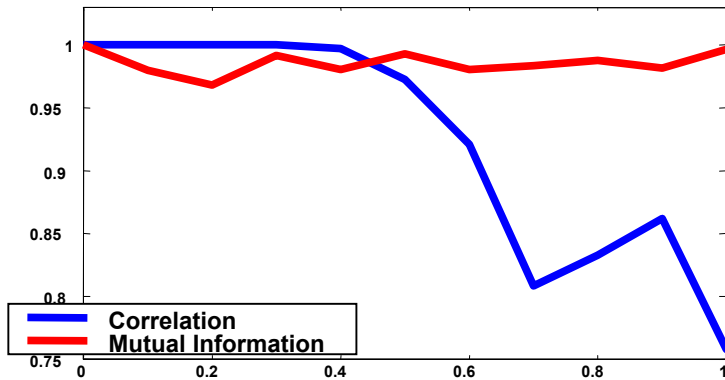


Both correlation and dependence degrade, but not at the same rate

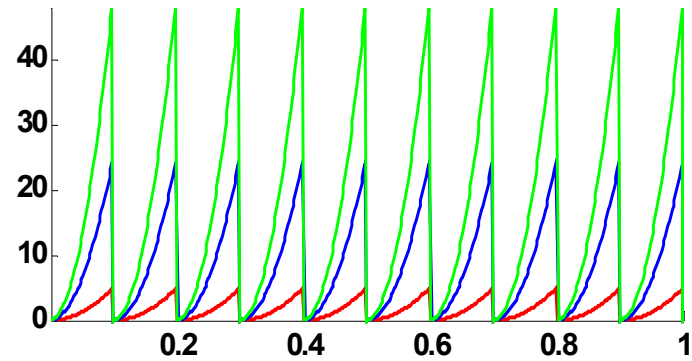


# Dispersive Medium

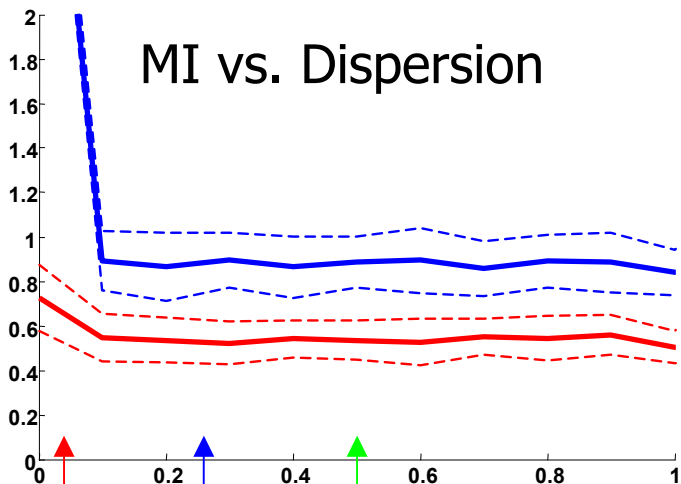
$P\{\text{correct association}\}$



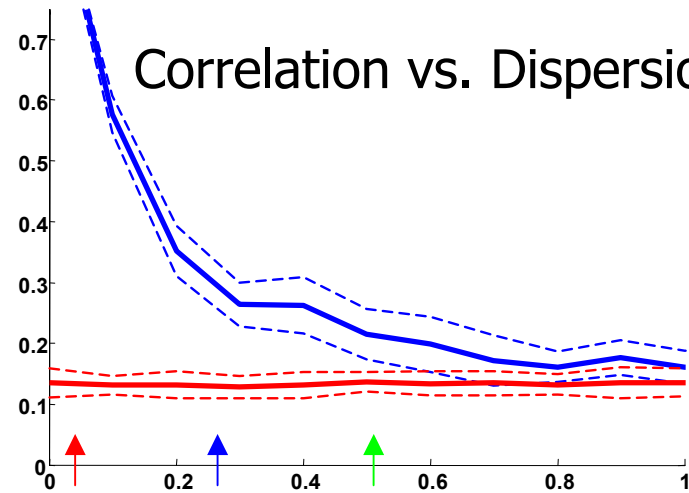
Phase response of media

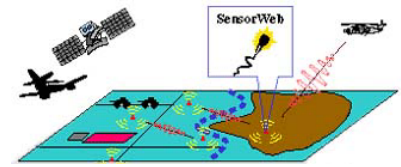


MI vs. Dispersion

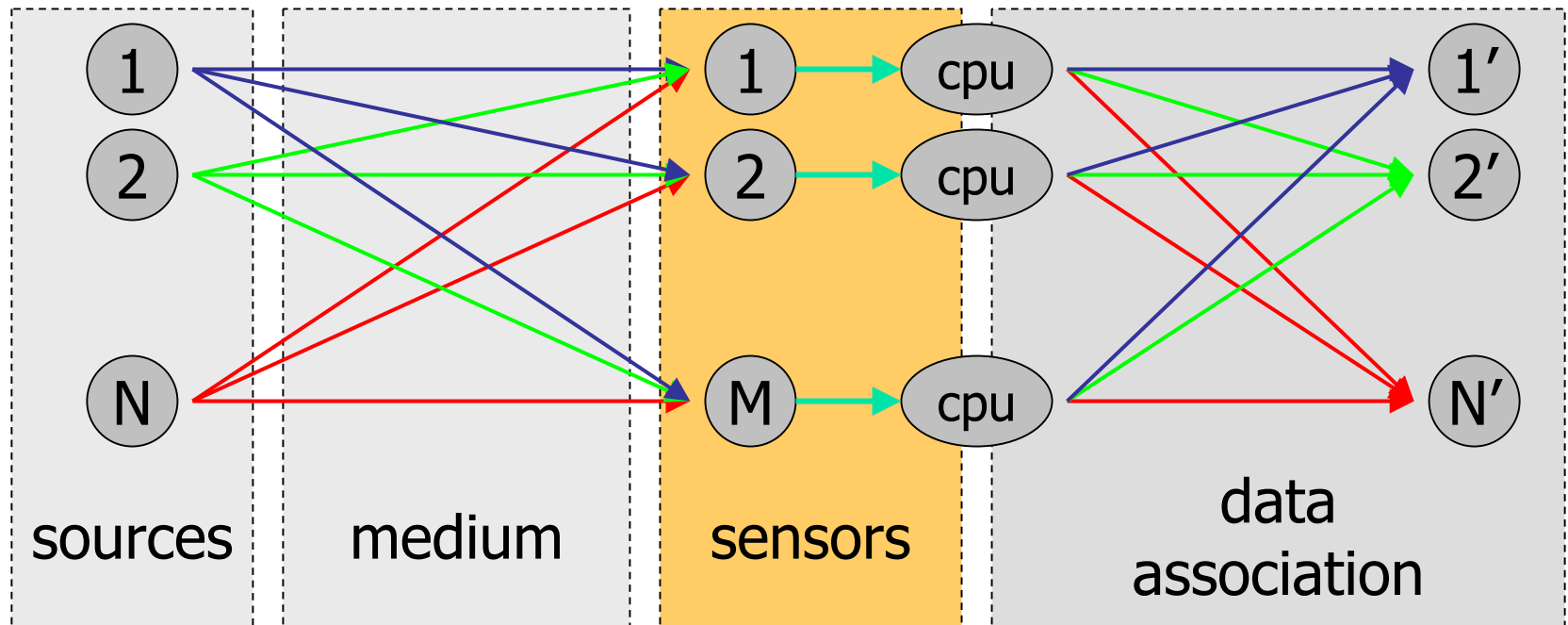


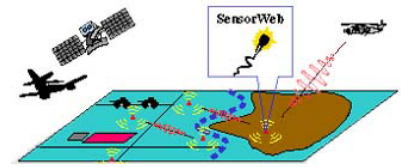
Correlation vs. Dispersion



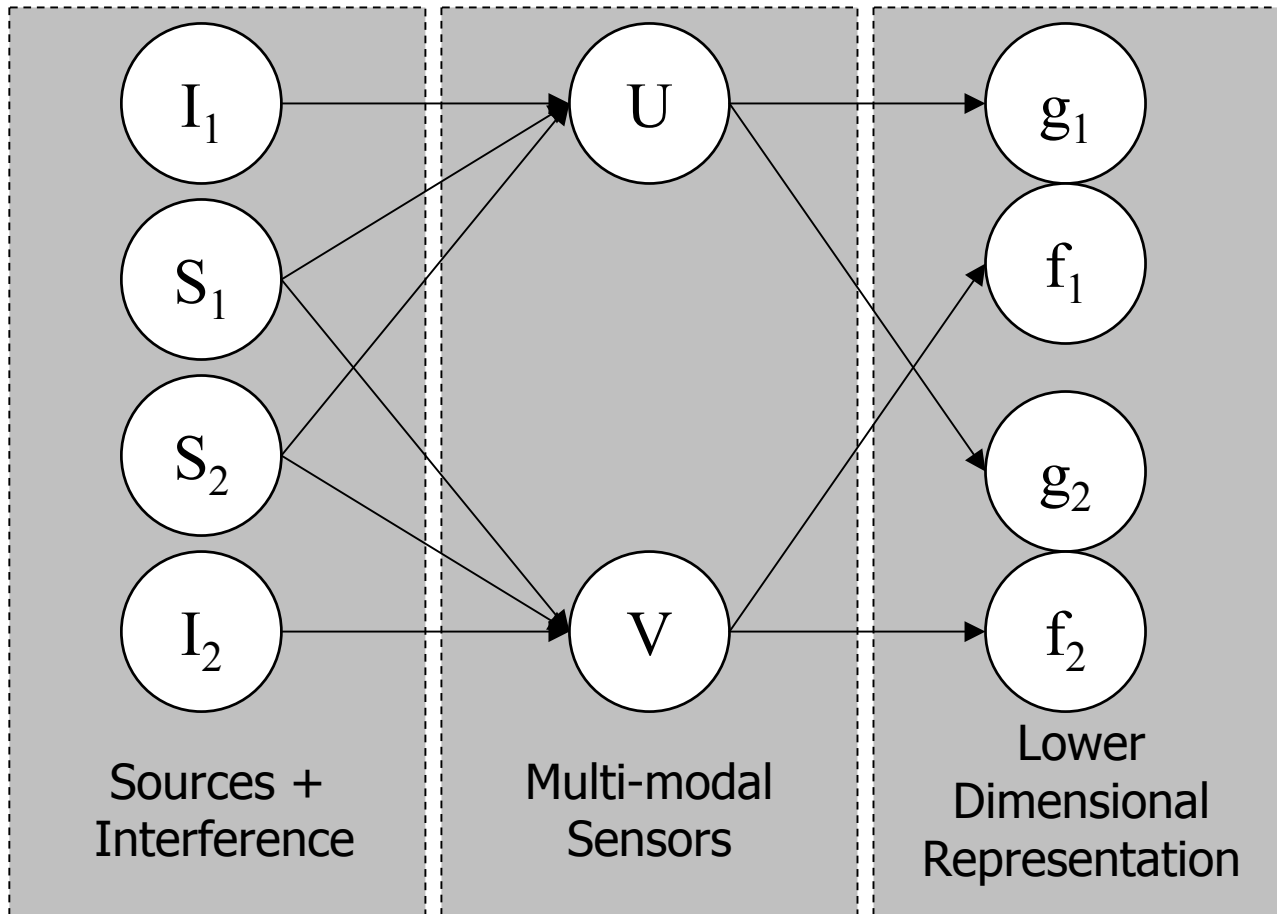


# Information Theoretic Sensor Fusion



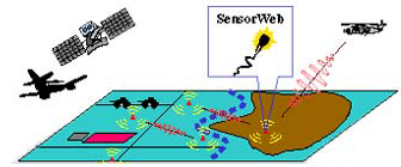


# Last Year



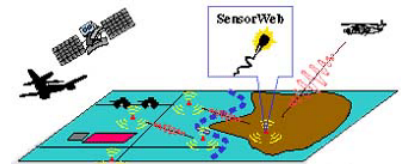
Maximizing  $I(g_1;f_1)$ ,  $I(g_2;f_2)$  and  $H([g_1,f_1],[g_2,f_2])$  recovers a representation of the sources up to a permutation (re: data association)



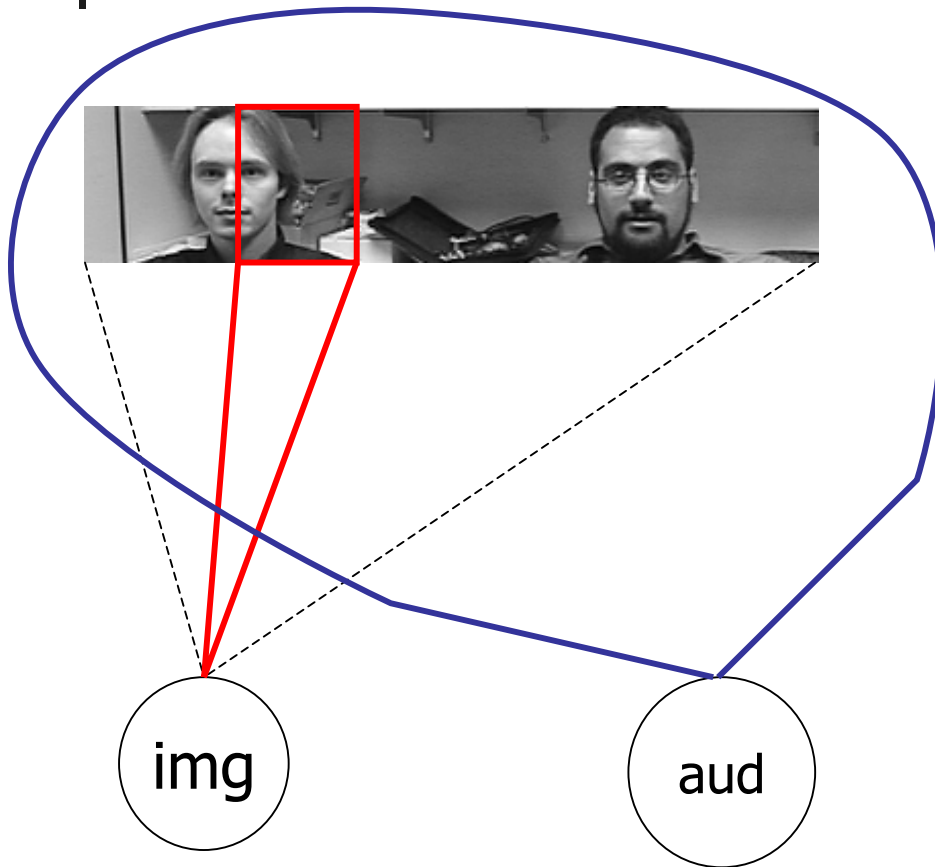


# Acoustically Steered Imaging Sensor

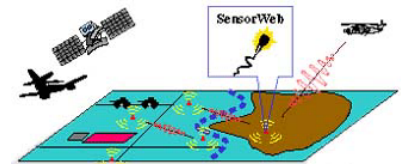
- Last time we presented our local fusion approach and justified it statistically.
- Here we focus on an application in the sensor domain considering a single narrow field of view imaging sensor (e.g. IR or video) guided by broad field of view acoustic sensors.



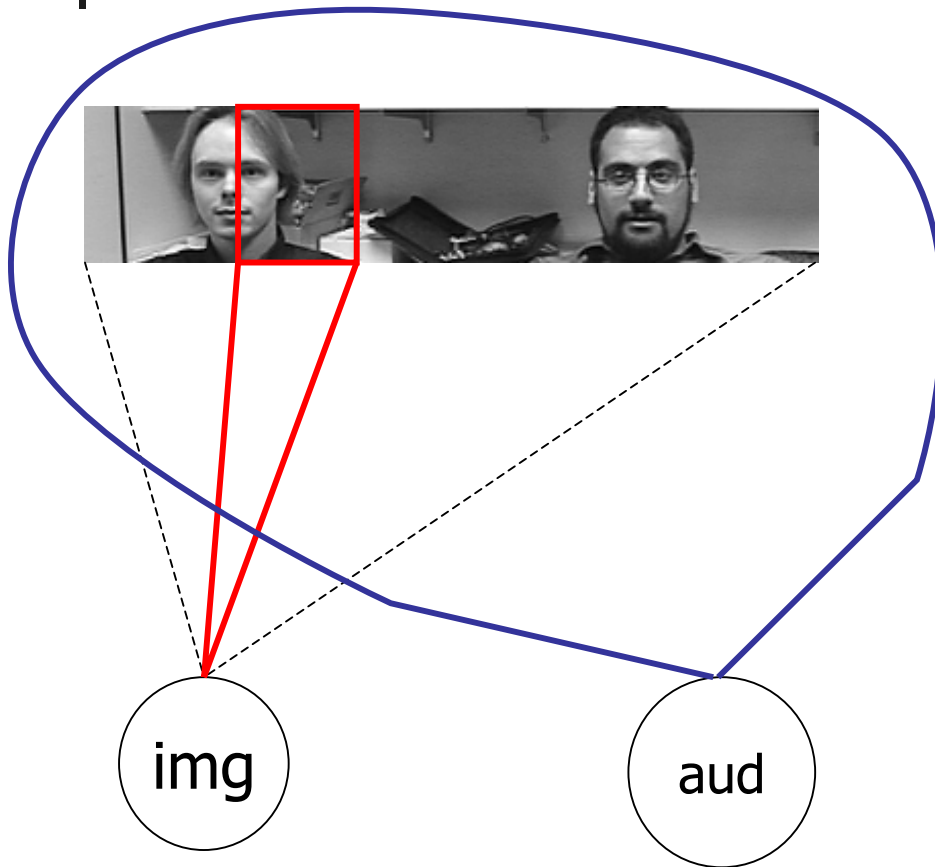
# acoustically steered imaging sensor



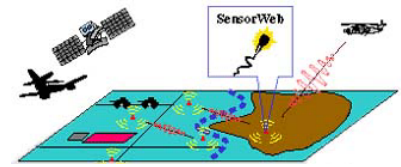
- Imaging sensor has a narrow field of view, but can be steered.
- Acoustic sensor has little directivity (broad field of view)



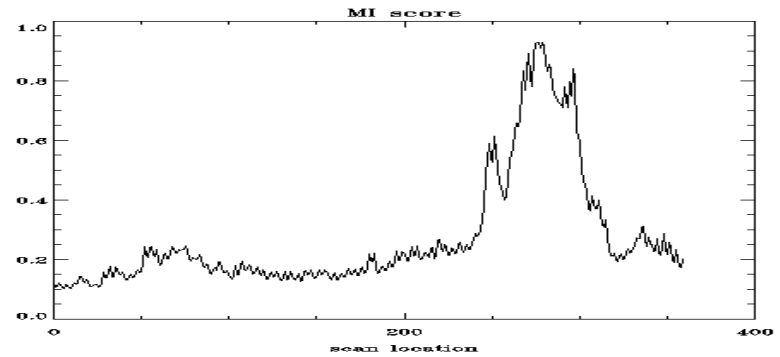
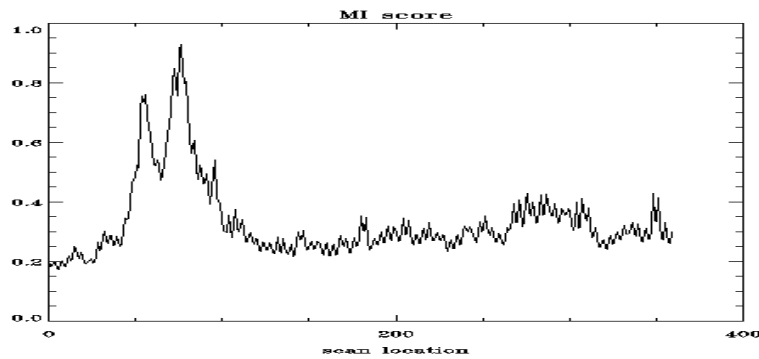
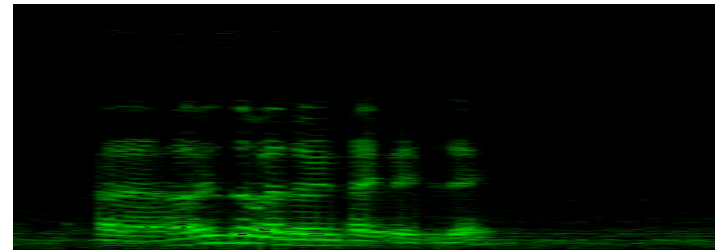
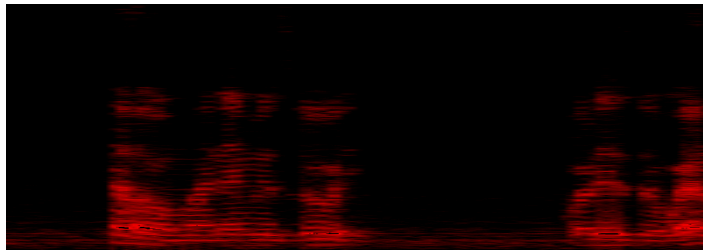
# acoustically steered imaging sensor

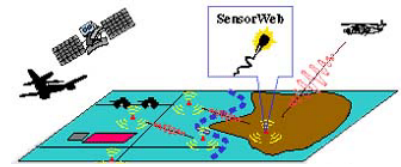


- Use local fusion to derive bearing to source when
  - One source is emitting an acoustic signal - opportunistic
  - Both sources emit acoustic signals simultaneously – local ambiguity



# Single source detection





# Multi Source Separation

